# CBCS SCHEME



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**15MAT41**

## Fourth Semester B.E. Degree Examination, June/July 2019
## Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



**Module-1**

1 a. Employ Taylor's series method, find y(0.1) considering upto third degree term if y(x) satisfies the equation $\frac{dy}{dx} = x - y^2$, y(0) = 1. **(05 Marks)**

b. Using Runge-Kutta method of fourth order, find y(0.1) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.1. **(05 Marks)**

c. Apply Milne's method to compute y(1.4) correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data : y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. **(06 Marks)**

**OR**

2 a. Use Taylor's series method to find y(4.1) given that $(x^2 + y)y' = 1$ and y(4) = 4. **(05 Marks)**

b. Find y at x = 0.8, given $y' = x - y^2$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Using Adams – Bashforth method. Apply the corrector formula. **(05 Marks)**

c. Using Modified Euler's method find y at x = 0.1 given $y' = 3x + \frac{y}{2}$ with y(0) = 1 taking h = 0.1. **(06 Marks)**

**Module-2**

3 a. Obtain the solution of the equation $2y'' = 4x + y'$ with initial conditions y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514 and y′(1) = 2, y′(1.1) = 2.3178, y′(1.2) = 2.6725, y′(1.3) = 3.0657 by computing y(1.4) applying Milne's method. **(05 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(05 Marks)**

c. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ **(06 Marks)**

**OR**

4 a. Given $y'' - xy' - y = 0$ with the initial conditions y(0) = 1, y′(0) = 0. Compute y(0.2) and y′(0.2) by taking h = 0.2 using Runge - Kutta method of fourth order. **(05 Marks)**

b. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ then, find the values of a, b, c, d. **(05 Marks)**

c. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]$$ **(06 Marks)**

## Module-3

5 a. State and prove Cauchy-Reimann equation in polar form. (05 Marks)

b. Discuss the transformation $w = z^2$. (05 Marks)

c. Find the bilinear transformation which maps the points $z = 1, i, -1$ into $w = 2, i, -2$. (06 Marks)

**OR**

6 a. Find the analytic function whose real part is

$$\frac{x^4 - y^4 - 2x}{x^2 + y^2}$$ (05 Marks)

b. State and prove Cauchy Integral formula. (05 Marks)

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z-2)} dz$ where c is the circle : $|z| = 3$ using Cauchy's Residue theorem. (06 Marks)

## Module-4

7 a. The probability function of a variate x is :

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| p(x) | 0 | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2 + k$ |

(i) Find k (ii) Evaluate $p(x < 6)$, $p(x \geq 6)$ and $p(3 < x \leq 6)$. (05 Marks)

b. Obtain mean and standard deviation of Binomial distribution. (05 Marks)

c. The joint distribution of two discrete variables x and y is $f(x, y) = (2x + y)$ where x and y are integers such that $0 \leq x \leq 2$; $0 \leq y \leq 3$.
Find : (i) Marginal distribution of x and y.
(ii) Are x and y independent. (06 Marks)

**OR**

8 a. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be
(i) less than 65 (ii) more than 75 (iii) between 65 and 75 [Given $\phi(1) = 0.3413$] (05 Marks)

b. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction. (05 Marks)

c. The joint distribution of the random variables X and Y are given. Find the corresponding marginal distribution. Also compute the covariance and the correlation of the random variables X and Y. (06 Marks)

| X \ Y | 1 | 3 | 9 |
|---|---|---|---|
| 2 | 1/8 | 1/24 | 1/12 |
| 4 | 1/4 | 1/4 | 0 |
| 6 | 1/8 | 1/24 | 1/12 |

## Module-5

9 a. Explain the terms: (i) Null hypothesis (ii) type-I and type-II errors (iii) Significance level **(05 Marks)**

b. In 324 throws of a six faced 'die', an odd number turned up 181 times. Is it reasonable to think that 'die' is an unbiased one? **(05 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball. **(06 Marks)**

**OR**

10 a. Find the unique fixed probability vector for the matrix

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

**(05 Marks)**

b. A random sample for 1000 workers in company has mean wage of Rs. 50 per day and standard deviation of Rs. 15. Another sample of 1500 workers from another company has mean wage of Rs. 45 per day and standard deviation of Rs. 20. Does the mean rate of wages varies between the two companies? **(05 Marks)**

c. A die is thrown 264 times and the number appearing on the face (x) follows the following frequency distribution.

| x | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| f | 40 | 32 | 28 | 58 | 54 | 60 |

Calculate the value of $\chi^2$. **(06 Marks)**

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