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14MAT11

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. If $x = \tan(\log y)$ then prove that $(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. Prove that the angle of intersection between the curves $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$ as $1/2\,(\pi + \cos^{-1}(1/3))$. **(07 Marks)**

c. Define the curvature and radius of curvature of a curve. Derive the expression for radius of curvature in polar form. **(07 Marks)**

OR

2 a. State Leibnitz theorem for n^{th} derive of product of two functions. Find the n^{th} derivative of $y = x^2 \log x$. **(06 Marks)**

b. Find the angle of intersection between the curves $\frac{2a}{r} = 1 + \cos\theta$ and $\frac{2a}{r} = 1 - \cos\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. **(07 Marks)**

Module-2

3 a. Expand logx in the powers of (x-1) upto and including $(x-1)^3$ and hence compute log (1.1). **(06 Marks)**

b. Define homogeneous function. Give suitable example.

If $u = \cos^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. **(07 Marks)**

c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3axy$. **(07 Marks)**

OR

4 a. Evaluate $\underset{x \to 0}{\text{Limit}} \left(\frac{e^x \sin x - x^2 - x}{x^2 + x\log(1-x)}\right)$. **(06 Marks)**

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. **(07 Marks)**

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find the value of $J\left(\frac{x, y, z}{r, \theta, \phi}\right)$. **(07 Marks)**

Module-3

5 a. Find a, b and c such that $\vec{F} = (axy - z^3)i + (bx^2 + z)j + (bxz^2 + cy)k$ is irrotational and find the scalar potential. **(06 Marks)**

b. Prove that $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x)} dx = \frac{\pi}{2}\log(1+a)$, a≥ 0 using differentiation under integration. **(07 Marks)**

c. Apply the general rule to trace the curve $r = a(1 + \cos\theta)$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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OR

6 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction of $2i - j - 2k$. **(06 Marks)**

b. Evaluate $\int_0^\alpha \frac{\log(1+\alpha x)}{1+x^2}dx$ and hence prove that $\int_0^1 \frac{\log(1+x)}{1+x^2} = \frac{\pi}{8}\log 2$. **(07 Marks)**

c. Apply the general rule to trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. **(07 Marks)**

Module-4

7 a. Establish the reduction formula for $\int \sin^m x \cos^n x\, dx$ and hence find $\int_0^{\pi/2} \sin^6 x \cos^4 dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{-(y\cos x + \sin y + y)}{\sin x + x\cos y + x}$. **(07 Marks)**

c. Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. **(07 Marks)**

OR

8 a. Establish the reduction formula of $\int \sin^n x\, dx$ and evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}dx$. **(06 Marks)**

b. Solve $x\frac{dy}{dx} + y = x^3y^6$. **(07 Marks)**

c. A 12 volt battery is connected to a series circuit in which the inductance is 1/2 Henry and the resistance is 10 ohms. Determine the current I if the initial current is zero. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ using elementary transformations. **(06 Marks)**

b. Use Gauss-Seidal iterative method to solve the system of equations 20x + y – 2z = 17, 3x + 20y – z = -18 and 2x – 3y + 20z = 25. (Use three iterations). Take initial values for {x, y, z} as {0, 0, 0}. **(07 Marks)**

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by Rayleigh's power method. (Use $x^{(0)} = [1\ \ 0\ \ 0]^T$. Take 5 iterations. **(07 Marks)**

OR

10 a. Solve by Gauss elimination method the system of equations,
x + y + z + t = 2, 2x – y + 2z – t = -5, 3x + 2y + 3z + 4t = 7 and x – 2y – 3z + 2t = 5. **(06 Marks)**

b. Reduce the quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ to canonical form. **(07 Marks)**

c. Test whether the transformation (x_1, x_2, x_3) to $(2x_1 + x_2 + x_3,\ x_1 + x_2 + 2x_3,\ x_1 - 2x_3)$ is non-singular. If so write the inverse transformation. **(07 Marks)**

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