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CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, June/July 2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express the complex number $\dfrac{(1+i)(1+3i)}{1+5i}$ in the form a + ib. **(05 Marks)**

b. Find the modulus and amplitude of $1+\cos\theta+i\sin\theta$. **(05 Marks)**

c. Show that $(a+ib)^n+(a-ib)^n = 2(a^2+b^2)^{n/2}\cos\left(n\tan^{-1}\left(\dfrac{b}{a}\right)\right)$ **(06 Marks)**

OR

2 a. If $\vec{A} = i-2j+3k$ and $\vec{B} = 2i+j+k$, find the unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. **(05 Marks)**

b. Show that the points $-6i+3j+2k$, $3i-2j+4k$, $5i+7j+3k$ and $-13i+17j-k$ are coplan. **(05 Marks)**

c. Prove that $\left[\vec{B}\times\vec{C},\ \vec{C}\times\vec{A},\ \vec{A}\times\vec{B}\right] = \left[\vec{A}\vec{B}\vec{C}\right]^2$ **(06 Marks)**

Module-2

3 a. Find the n^{th} derivative of $\dfrac{x}{(x-1)(2x+3)}$. **(05 Marks)**

b. Find the angle of intersection of the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$. **(05 Marks)**

c. Obtain the Maclourin series expansion of the function sin x upto the term containing x^4. **(06 Marks)**

OR

4 a. Show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y} = 2u\log u$ where $\log u = \dfrac{x^3+y^3}{3x+4y}$. **(05 Marks)**

b. If $u = f(x-y, y-z, z-x)$ prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z} = 0$. **(05 Marks)**

c. If $u = x+3y^2-z^3$, $v = 4x^2yz$, $w = 2z^2-xy$, evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0). **(06 Marks)**

Module-3

5 a. Obtain the reduction formula for $\int \sin^n x\,dx$. Hence evaluate $\int_0^{\pi/2}\sin^n x\,dx$. **(05 Marks)**

b. Evaluate $\displaystyle\int_0^{\infty}\frac{x^6}{(1+x^2)^7}dx$. **(05 Marks)**

c. Evaluate $\displaystyle\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$. **(06 Marks)**

OR

6 a. Evaluate $\int_0^{2a}\int_0^{x^2/4a} xy\,dy\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1\int_0^1\int_0^1 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

c. Evaluate $\int_0^a \frac{x^7 dx}{\sqrt{a^2-x^2}}$ by using reduction formula. **(06 Marks)**

Module-4

7 a. A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+3$ where t is the time. Find the components of velocity and acceleration at $t=1$ in the direction of $i+j+3k$. **(05 Marks)**

b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ where $\vec{F}=\operatorname{grad}(x^3+y^3+z^3-3xyz)$. **(05 Marks)**

c. Prove that $\operatorname{div}(\operatorname{curl}\vec{F})=0$. **(06 Marks)**

OR

8 a. Find the directional derivative of $f(x,y,z)=xy^3+yz^3$ at (2, −1, 1) in the direction of $i+2j+2k$. **(08 Marks)**

b. Prove that $\nabla^2\left(\frac{1}{r}\right)=0$ where $r=\sqrt{x^2+y^2+z^2}$. **(08 Marks)**

Module-5

9 a. Solve $(x^2-y^2)dx-xy\,dy=0$. **(05 Marks)**

b. Solve $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+(x+\log x-x\sin y)dy=0$. **(05 Marks)**

c. Solve $\frac{dy}{dx}-\frac{y}{1+x}=e^{3x}(x+1)$. **(06 Marks)**

OR

10 a. Solve $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$. **(08 Marks)**

b. Solve $(3y+2x+4)dx-(4x+6y+5)dy=0$. **(08 Marks)**

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