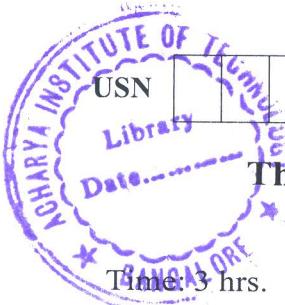


CBCS SCHEME



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Date.....

Time: 3 hrs.

15MATDIP31

Third Semester B.E. Degree Examination, June/July 2019

Additional Mathematics – I

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Express the complex number $\frac{(1+i)(1+3i)}{1+5i}$ in the form $a + ib$. (05 Marks)
- b. Find the modulus and amplitude of $1 + \cos \theta + i \sin \theta$. (05 Marks)
- c. Show that $(a+ib)^n + (a-ib)^n = 2(a^2 + b^2)^{n/2} \cos\left(n \tan^{-1}\left(\frac{b}{a}\right)\right)$ (06 Marks)

OR

- 2 a. If $\vec{A} = i - 2j + 3k$ and $\vec{B} = 2i + j + k$, find the unit vector perpendicular to both \vec{A} and \vec{B} . (05 Marks)
- b. Show that the points $-6i + 3j + 2k$, $3i - 2j + 4k$, $5i + 7j + 3k$ and $-13i + 17j - k$ are coplan. (05 Marks)
- c. Prove that $\left[\vec{B} \times \vec{C}, \vec{C} \times \vec{A}, \vec{A} \times \vec{B} \right] = \left[\vec{A} \vec{B} \vec{C} \right]^2$ (06 Marks)

Module-2

- 3 a. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. (05 Marks)
- b. Find the angle of intersection of the curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$. (05 Marks)
- c. Obtain the Maclourin series expansion of the function $\sin x$ upto the term containing x^4 . (06 Marks)

OR

- 4 a. Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$ where $\log u = \frac{x^3 + y^3}{3x + 4y}$. (05 Marks)
- b. If $u = f(x-y, y-z, z-x)$ prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (05 Marks)
- c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$. (06 Marks)

Module-3

- 5 a. Obtain the reduction formula for $\int \sin^n x dx$. Hence evaluate $\int_0^{\pi/2} \sin^n x dx$. (05 Marks)
- b. Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^7} dx$. (05 Marks)
- c. Evaluate $\iiint_{-1 \leq x \leq z \leq x+z} (x+y+z) dx dy dz$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and / or equations written e.g., $42+8=50$, will be treated as malpractice.

OR

- 6 a. Evaluate $\int_0^{2a} \int_0^{x^2/4a} xy dy dx$. (05 Marks)
- b. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z) dx dy dz$. (05 Marks)
- c. Evaluate $\int_0^a \frac{x^7 dx}{\sqrt{a^2 - x^2}}$ by using reduction formula. (06 Marks)

Module-4

- 7 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the components of velocity and acceleration at $t = 1$ in the direction of $i + j + 3k$. (05 Marks)
- b. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. (05 Marks)
- c. Prove that $\text{div}(\text{curl } \vec{F}) = 0$. (06 Marks)

OR

- 8 a. Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at $(2, -1, 1)$ in the direction of $i + 2j + 2k$. (08 Marks)
- b. Prove that $\nabla^2 \left(\frac{1}{r} \right) = 0$ where $r = \sqrt{x^2 + y^2 + z^2}$. (08 Marks)

Module-5

- 9 a. Solve $(x^2 - y^2)dx - xy dy = 0$. (05 Marks)
- b. Solve $\left[y \left(1 + \frac{1}{x} \right) + \cos y \right] dx + (x + \log x - x \sin y) dy = 0$. (05 Marks)
- c. Solve $\frac{dy}{dx} - \frac{y}{1+x} = e^{3x}(x+1)$. (06 Marks)

OR

- 10 a. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. (08 Marks)
- b. Solve $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$. (08 Marks)
