USN | | | | | | | | | | | **10CS34**

Third Semester B.E. Degree Examination, June/July 2019

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. For any two sets A and B, prove the following $A-(A-B)=A\cap B$. **(05 Marks)**

b. Thirty cars are assembled in a factory. The options available are a music system, an air conditioner and power windows. It is known that 15 of the cars have music systems, 8 have air conditioners and 6 have power windows. Further, 3 have all options. Determine at least how many cars do not have any option at all. **(05 Marks)**

c. The probability that an integrated circuit will have defective etching is 0.12, the probability that it will have crack defect is 0.29, and the probability that it will have both defects is 0.07. What is the probability that a newly manufactured chip will have
(i) an etching or crack defect? (ii) neither defect? **(05 Marks)**

d. Define countable set. Show that the set of all integers is countable. **(05 Marks)**

2 a. For any two propositions p and q, prove by using truth tables that $(p\vee q)\wedge(p\leftrightarrow q)$ is a contradiction. **(05 Marks)**

b. Prove the following logical equivalence without using truth table:
$[p\vee q\vee(\neg p\wedge\neg q\wedge r)]\Leftrightarrow(p\vee q\vee r)$ **(05 Marks)**

c. Express the following in terms of only NAND and only Nor connectives: $p\wedge q$. **(05 Marks)**

d. Test whether the following argument is valid: **(05 Marks)**

$$\begin{array}{l} p\to q \\ r\to s \\ \underline{p\vee r} \\ \therefore q\vee s \end{array}$$

3 a. For the universe of all integers, let p(x) : x > 0,
q(x): x is even r(x): x is a perfect square
s(x): x is divisible by 3 t(x): x is divisible by 7.
Express each of the following symbolic statements in words and indicate its truth value.
i) $\forall x, [r(x)\to p(x)]$ ii) $\exists x, [s(x)\wedge\neg q(x)]$
iii) $\forall x, [\neg r(x)]$ iv) $\forall x, [r(x)\vee t(x)]$ **(06 Marks)**

b. Find whether the following argument is valid:

No Engineering student of first or second semester studies logic
Anil is an engineering student who studies logic.
∴ Anil is not in second semester
(07 Marks)

c. Give (i) a direct proof (ii) an indirect proof and (iii) proof by contradiction for the statement: "If n is an odd integer then n + 11 is an even integer". **(07 Marks)**

4 a. Using mathematical induction, prove that $n! \geq 2^{n-1} \forall n \geq 1$. **(05 Marks)**

b. Obtain the recursive definition for the sequence $\{a_n\}$ in each of the following:
(i) $a_n = n(n+2)$ (ii) $a_n = 3n+7$ **(05 Marks)**

c. For the Fibonacci sequence $F_0, F_1, F_2, \ldots$ Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(05 Marks)**

d. Given $F_n = F_{n-1} + F_{n-2}$ $\forall n \geq 2$ with $F_0 = 0$, $F_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ $\forall n \geq 2$ with $L_0 = 2$, $L_1 = 1$. Prove that $L_n = F_{n-1} + F_{n+1}$ for all positive integers n. **(05 Marks)**

PART – B

5 a. For an non-empty sets A, B, C, prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(06 Marks)**

b. Let A = {1, 2, 3}, R and S be the relations on A whose matrices are $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find the composite relations RoS, SoR, R^2, S^2 and their matrices. **(07 Marks)**

c. Let R be a relation on a set A. Prove that:
i) R is reflexive if and only if $\overline{R}$ is irreflexive.
ii) If R is symmetric, so are R^C and $\overline{R}$. **(07 Marks)**

6 a. For any sets A, B, C contained in a universal set U. Prove that (i) $f_{\overline{A}}(x) = 1 - f_A(x)$ (ii) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x) f_B(x)$. **(06 Marks)**

b. Let $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set A = {1, 2, 3, 4, 5, 6}.
i) Write P as a product of disjoint cycles.
ii) Compute P^{-1}.
iii) Find the smallest positive integer K such that $P^K = I_A$. **(07 Marks)**

c. Show that (P(A), $\subseteq$) is a Lattice, given A = {1, 2, 3}. **(07 Marks)**

7 a. Show that A = {2, 4, 6, 8} under $\otimes_{10}$ is an Abelian group. **(07 Marks)**

b. State and prove Lagrange's theorem. **(06 Marks)**

c. Define Homomorphism and Isomorphism of a group. Prove that the function $f : R \to R^+$ defined as $f(x) = e^x \forall x \in R$ is an isomorphism. **(07 Marks)**

8 a. Let $C \subseteq Z_2^n$ be a group code for a parity-check matrix H and Let $x, y \in Z_2^n$. Prove that x and y are in the same coset of C if and only if $H[x]^T = H[y]^T$. **(06 Marks)**

b. Prove that the set Z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a Ring. **(07 Marks)**

c. Prove that Z_n is a field if and only if n is a prime. **(07 Marks)**

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