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10CS42

Fourth Semester B.E. Degree Examination, June/July 2019
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Prove that in every graph the number of vertices of odd degree is even. **(05 Marks)**

b. Define isomorphism of two graphs. Determine whether the following graphs are isomorphic or not. **(05 Marks)**

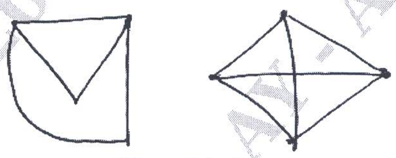

Fig. Q1 (b)

c. Define a complete graph. In the complete graph with n vertices, where n is an odd number ≥ 3, show that there are $\frac{(n-1)}{2}$ edge disjoint Hamilton cycles. **(05 Marks)**

d. Define a regular graph. Draw a graph which has 10-vertices and 15 edges which should be 3-regular graph. **(05 Marks)**

2 a. Define planar graph. Examine if the following graphs are planar : (i) K_4 (ii) $K_{3,3}$. **(05 Marks)**

b. Define chromatic number of a graph. Find the chromatic polynomial $P(G, \lambda)$ for the following graph: **(05 Marks)**

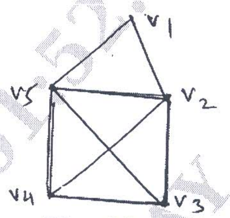

Fig. Q2 (b)

c. Find the geometrical dual of the following graph. Write down any four observations of the graph Fig. Q2 (c) and its dual. **(05 Marks)**

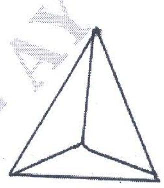

Fig. Q2 (c)

d. Define Hamiltonian and Euler graphs. Compare the two graphs. **(05 Marks)**

3 a. Define a tree. Prove that the tree $G = (V, t)$ with P vertices has P – 1 edges. **(07 Marks)**

b. Define a spanning tree of a graph. Find all the spanning trees of the following graph, Fig. Q3 (b). **(07 Marks)**

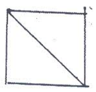

Fig. Q3 (b)

c. Define: (i) Rooted tree (ii) Balanced tree (iii) Prefix code. Give an example for each. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 a. Define : (i) Cutset (ii) Edge connectivity (iii) Vertex connectivity. Give one example for each. **(06 Marks)**

b. For the network shown in Fig. Q4 (b), find the capacities of all the cutsets between the vertices a and d and hence determine the maximum flow between a and b. **(06 Marks)**

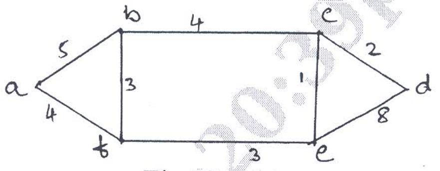

Fig. Q4 (b)

c. Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown in Fig. Q4 (c). **(08 Marks)**

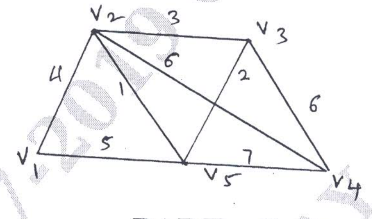

Fig. Q4 (c)

PART – B

5 a. Find the number of ways of giving 10 identical gift boxes to six persons A, B, C, D, E, F in such a way that the total number of boxes given to A and B together does not exceed 4. **(06 Marks)**

b. Define derangement. These are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that at least one letter goes to the right person. **(06 Marks)**

c. Define Catelan numbers. In how many ways can one travel in the xy plane from (0, 0) to (0, 8) using the moves R(x+1, y) and R(x, y+1) if the path taken may touch but never rise above the line $y = x$? Draw two such paths in the xy plane. **(08 Marks)**

6 a. In how many ways can the 26 letters of the alphabet be permitted so that none of the patterns spin, game, path or not occurs? **(06 Marks)**

b. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish to have apple the boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased. **(08 Marks)**

c. How many integers between 1 and 300 (inclusive) are,
(i) divisible by 5, 6, 8? (ii) divisible by none of 5, 6, 8? **(06 Marks)**

7 a. Find the coefficients of x^{18} in the following products:

(i) $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + x^5 .....)^5$.

(ii) $(x + x^3 + x^5 + x^7 + x^9)(x + 2x^4 + 3x........)^3$ **(06 Marks)**

b. A bag contains a large number of red, green, white and black marbles, with at least 24 of each color. In how many ways can one select 24 of these marbles, so that these are even numbers of white marbles and at least six black marbles? **(07 Marks)**

c. Using generating function, find the number of partitions of $n = 6$. **(07 Marks)**

8 a. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42. Hence find the general term of the sequence. **(06 Marks)**

b. Find the generating function for the Fibonacci sequence (Fn) and hence obtain an expression for Fn. **(07 Marks)**

c. Solve the following recurrence relations by the method of generating functions, $a_n = 2(a_{n-1} - a_{n-2})$ where $n \geq 2$ and given that $a_0 = 1$ and $a_1 = 2$. **(07 Marks)**

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