**CBCS SCHEME**

USN                      **15CS36**

# Third Semester B.E. Degree Examination, June/July 2019

# Discrete Mathematical Structures

Time: 3 hrs.          Max. Marks: 80

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

## Module-1

1 a. Simplify the switching network shown in Fig Q1(a)

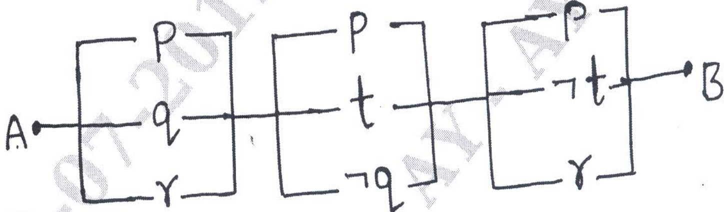


Fig Q1(a) **(08 Marks)**

b. Give a direct proof of the statement "If n is an odd integer then $n^2$ is also an odd integer". **(04 Marks)**

c. Let p(x), q(x) and r(x) be open statements that are defined for the given universe. Show that the argument.

$\forall x, [p(x) \rightarrow q(x)]$

$\underline{\forall x, [q(x) \rightarrow r(x)]}$

$\therefore \exists x, [p(x) \rightarrow r(x)]$ is valid **(04 Marks)**

**OR**

2 a. Define tautology, prove that for any proposition p, q, r the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow q)$ is a tautology using truth table. **(05 Marks)**

b. Show that RVS follows logically form the premises CVD, CVD→¬H, ¬H → (A ∧ ¬B) and (A ∧ ¬B) → (RVS). **(04 Marks)**

c. Using rules of inference shows that the following argument is valid.

$\forall x, [p(x) \vee q(x)] \wedge \exists x, \neg p(x) \wedge$

$\forall x, [\neg q(x) \vee r(x)] \wedge \forall x, [s(x) \rightarrow \neg r(x)]$

$\therefore \exists x, \neg S(x)$ **(07 Marks)**

## Module-2

3 a. Prove by mathematical induction that, for all integers n ≥ 1, $1 + 2 + 3 + \ldots + n = \frac{1}{2}n(n+1)$. **(06 Marks)**

b. The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Evaluate $F_2$ to $F_{10}$. **(04 Marks)**

c. In the word S, O, C, I, O, L, O, G, I, C, A, L.

i) How many arrangements are there for all letters?

ii) In how many of these arrangements all vowels are adjacent? **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

**OR**

4 a. Obtain the recursive definition for the sequence $\{a_n\}$ in each of the following cases.
(i) $a_n = 5n$ (ii) $a_n = 6^n$ (iii) $a_n = n^2$ **(06 Marks)**

b. Find the coefficient of
i) $x^9 y^3$ in the expansion fo $(2x - 3y)^{12}$
ii) $x^{12}$ in the expansion of $x^3 (1 - 2x)^{10}$ **(04 Marks)**

c. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with atleast 3 spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message? **(06 Marks)**

## Module-3

5 a. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$ determine $f(0)$, $f(-1)$, $f^{-1}(0)$, $f^{-1}(+3)$, $f^{-1}([-5, 5])$ **(08 Marks)**

b. Define an equivalence relation. Write the partial order relation for the positive divisors of 36 and write its Hasse diagram (HASSE). **(08 Marks)**

**OR**

6 a. Consider the function $f : R \to R$ defined by $f(x) = 2x + 5$. Let a function $g : R \to R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is an inverse of f. **(03 Marks)**

b. State Pigeonhole principle. Let ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that atleast two of their points are such that the distance between them is less than ½ cm. **(05 Marks)**

c. If $A = \{1, 2, 3, 4\}$, R and S are relations on A defined by $R = \{(1, 2), (1, 3), (2, 4), (4, 4)\}$ $S = \{(1, 1), (1, 2), (1,3), (1, 4), (2, 3), (2, 4)\}$ find RoS, SoR, $R^2$, $S^2$ and write down their matrices. **(08 Marks)**

## Module-4

7 a. Find the number of derangements of 1, 2, 3, 4 list all such derangements. **(04 Marks)**

b. Determine the number of integers between 1 and 300 (inclusive) which are divisible by exactly 2 of 5, 6, 8. **(06 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day? **(06 Marks)**

**OR**

8 a. Five teachers $T_1$, $T_2$, $T_3$, $T_4$, $T_5$ are to be made class teachers for 5 classes $C_1$, $C_2$, $C_3$, $C_4$, $C_5$ one teacher for each class $T_1$ and $T_2$ donot wish become the class teachers for $C_1$ or $C_2$, $T_3$ and $T_4$ for $C_4$ or $C_5$ and $T_5$ for $C_3$ or $C_4$ or $C_5$. In how many ways can teachers be assigned the work (without displeasing any teacher)? **(08 Marks)**

b. Solve the recurrence relation,
$a_n = 2(a_{n-1} - a_{n-2})$, where $n \ge 2$ and $a_0 = 1$, $a_1 = 2$. **(08 Marks)**

## Module-5

9 a. Prove that the undirected graph G = (V, E) has an Euler circuit if and only if G is connected and every vertex in G has even degree. **(08 Marks)**

b. Define binary rooted tree and Balanced tree. Draw all the spanning trees of the graph shown in Fig 9(b)

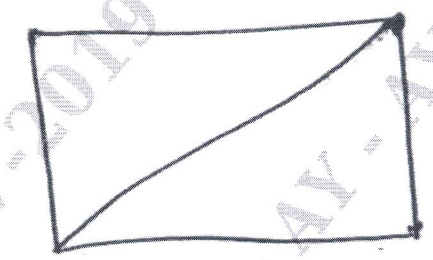

Fig Q9(b) **(08 Marks)**

**OR**

10 a. Define, with an example for each Regular graph, complement of a graph, Euler trail and Euler circuit and complete graph. **(10 Marks)**

b. Apply Merge sort to the list
6, 2, 7, 3, 4, 9, 5, 1, 8 **(06 Marks)**

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