

## Third Semester B.E. Degree Examination, June/July 2019 Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

### Module-1

- 1 a. Define tautology. Verify that  $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$  is a tautology. (06 Marks)
- b. If statement  $q$  has truth value 1, determine all truth value assignments for the primitive statements  $p, r, s$  for which the truth value of the statement :  
 $(q \rightarrow [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \rightarrow (\neg r \wedge q)]$  is 1. (04 Marks)
- c. Establish the following logical equivalence :  
 i)  $p \vee q \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow p \vee q \vee r$   
 ii)  $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q.$  (10 Marks)

### OR

- 2 a. Establish the validity of following arguments :
- |   |  |
|---|--|
| $\begin{array}{l} r \rightarrow t \\ \neg t \\ \hline \therefore p \end{array}$ | $\begin{array}{l} \text{ii) } u \rightarrow r \\ (r \wedge s) \rightarrow (p \vee t) \\ q \rightarrow (u \wedge s) \\ \neg t \\ \hline q \\ \hline \therefore p \end{array}$ |
|---|--|
- (08 Marks)
- b. Let  $p(x), q(x)$  and  $r(x)$  be the following open statements :  
 $p(x) : x^2 - 7x + 10 = 0$     $q(x) : x^2 - 2x - 3 = 0$     $r(x) < 0$ .  
 Determine truth or falsity of following statements, where universe is all integers. If a statement is false, provide a counter example.  
 i)  $\forall x [p(x) \rightarrow \neg r(x)]$    ii)  $\forall x [q(x) \rightarrow r(x)]$   
 iii)  $\exists x [q(x) \rightarrow r(x)]$    iv)  $\exists x [p(x) \rightarrow r(x)]$ . (08 Marks)
- c. Prove that for all integers 'k' and 'l', if 'k' and 'l' are both even, then  $k + l$  is even and  $kl$  is even by direct proof. (04 Marks)

### Module-2

- 3 a. Define well ordering principle and prove the following by mathematical induction :
- i)  $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$
- ii)  $1 * 3 + 2 * 4 + 3 * 5 + \dots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$ . (12 Marks)
- b. Find the coefficients of :  
 i.  $x^9 y^3$  in the expansion of  $(2x - 3y)^{12}$   
 ii.  $a^2 b^3 c^2 d^5$  in the expansion of  $(a + 2b - 3c + 2d + 5)^{16}$ . (08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8=50, will be treated as malpractice.

OR

- 4 a. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in following situations,
- There is no restriction on the choice
  - Two particular persons will not attend separately
  - Two particular persons will not attend together.
- (06 Marks)
- b. How many arrangements are there for all letters in word SOCIOLOGICAL? In how many of these arrangements all vowels are adjacent.
- (06 Marks)
- c. For the Fibonacci sequence  $F_0, F_1, F_2 \dots$  prove that  $F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right]$ .
- (08 Marks)

Module-3

- 5 a. Let  $A = \{1, 2, 3, 4\}$  and  $B = \{1, 2, 3, 4, 5, 6\}$ .
- How many functions are there from A to B?
  - How many of these are one to one?
  - How many are onto?
  - How many functions are there from B to A?
  - How many of these are onto?
  - How many are one to one?
- (06 Marks)
- b. A computer operator is given a magnetic tape that contains 500,000 words of four or fewer lowercase letters. Can it be that the 500,000 words are all distinct?
- (06 Marks)
- c. Let  $f, g, h : \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = x^2$ ,  $g(x) = x + 5$  and  $h(x) = \sqrt{x^2 + 2}$ . Show that  $(hog) \circ f = ho(gof)$ .
- (08 Marks)

OR

- 6 a. Let  $A = \{1, 2, 3, 6, 9, 18\}$  and define R on A by  $xRy$  if "x divides y", Draw the Hasse diagram for the poset (A, R). Also write the matrix of relation.
- (08 Marks)
- b. Consider Poset whose Hasse diagram is given below. Consider  $B = \{3, 4, 5\}$ . Find upper and lower bounds of B, least upper bound and greatest lower bound of B.
- (04 Marks)
- (Ref. Fig.Q6(b)).

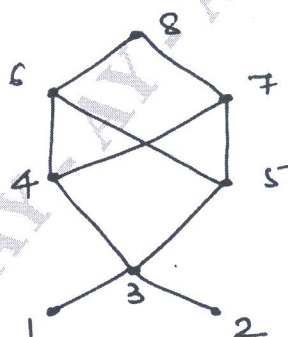


Fig.Q6(b)

- c. Let  $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$  and define R on A by  $(x_1, y_1) R (x_2, y_2)$  if  $x_1 + y_1 = x_2 + y_2$ .
- Verify that R is an equivalence relation on A
  - Determine equivalence classes  $[(1, 3)]$ ,  $[(2, 4)]$  and  $[(1, 1)]$
  - Determine partition of A induced by R.
- (08 Marks)



**Module-4**

- 7 a. In how many ways can the 26 letters of English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? (08 Marks)
- b. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that atleast one letter gets to right person. (04 Marks)
- c. Four persons  $P_1, P_2, P_3, P_4$  who arrive late for a dinner party find that only one chair at each of five table  $T_1, T_2, T_3, T_4$  and  $T_5$  is vacant.  $P_1$  will not sit at  $T_1$  or  $T_2$ ,  $P_2$  will not sit at  $T_2, P_3$  will not sit at  $T_3$  or  $T_4$  and  $P_4$  will not sit at  $T_4$  or  $T_5$ . Find the number of ways they can occupy the vacant chairs. (08 Marks)

OR

- 8 a. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42, . . . . Hence find the general term of the sequence. (10 Marks)
- b. If  $a_0 = 0, a_1 = 1, a_2 = 4$  and  $a_3 = 37$  satisfy the recurrence relation  $a_{n+2} + ba_{n+1} + ca_n = 0$  for  $n \geq 0$ , determine the constants b and c and then solve the relation for  $a_n$ . (10 Marks)

**Module-5**

- 9 a. Merge sort the list  $-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3$ . (06 Marks)
- b. Determine whether the following graphs are isomorphic or not. (06 Marks)

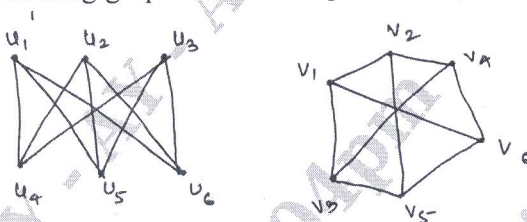


Fig.Q9(b)

- c. Define the following with an example to each.  
 i) Simple graph ii) Complete graph iii) Regular graph iv) Spanning sub graph v) Induced subgraph vi) Complete Bipartite graph vii) Tree viii) Complement of graph. (08 Marks)

OR

- 10 a. Define trail, circuit, path, cycle. In the graph shown below determine : [Ref.Q10(a)]  
 i. a walk from b to d that is not a trail  
 ii. b-d trail that is not a path  
 iii. a path from b to d  
 iv. a closed walk from b to b that is not a circuit  
 v. a circuit from b to b that is not cycle  
 vi. a cycle form b to b. (10 Marks)

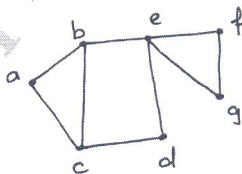


Fig.Q10(a)

- b. Define optimal tree and construct an optimal tree for a given set of weights  $\{4, 15, 25, 5, 8, 16\}$ . Hence find the weight of optimal tree. (06 Marks)
- c. Prove that in a graph. The sum of degrees of all vertices is an even number and is equal to twice the number of edges in the graph. (04 Marks)

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