USN | | | | | | | | | | **13MCA12**

First Semester MCA Degree Examination, June/July 2015
Discrete Mathematics Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Let p, q and r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions:
i) $(p \vee q) \vee r$ ii) $(p \wedge q) \wedge r$ iii) $(p \wedge q) \rightarrow r$
iv) $p \rightarrow (q \wedge r)$ v) $p \wedge (r \rightarrow q)$ vi) $p \rightarrow (q \rightarrow (\neg r))$ **(06 Marks)**

b. By employing laws of logic, prove the following:
i) $[(p \vee q) \wedge (p \vee \neg q)] \vee q \Leftrightarrow p \vee q$ ii)
ii) $(p \rightarrow) \wedge \{\neg q \wedge (r \vee \neg q)\} \Leftrightarrow \neg (q \vee p)$. **(07 Marks)**

c. Test whether the following is a valid argument: I will get grated A in this course or I will not graduate. If I do not graduate, I will join the army.

$$\frac{\text{I got grade A}}{\therefore \text{I will not join the army}}.$$ **(07 Marks)**

2 a. For the universe of integers, let
P(x): x > 0, q(x): x is even, r(x): x is a perfect square, s(x): x is divisible by 3, t(x): x is divisible by 7. Express each of the following symbolic statements in words and indicate its truth value:
i) $\forall_x, \{r(x) \rightarrow b(x)\}$ ii) $\exists_x, \{s(x) \wedge \neg q(x)\}$ iii) $\forall_x, \{r(x) \vee t(x)\}$. **(06 Marks)**

b. Find whether the following argument is valid:
NO MCA student of first or second semester studies logic.

$$\frac{\text{Anil is an MCA student who studies logic}}{\therefore \text{Anil is not in second semester}}.$$ **(07 Marks)**

c. Give: i) a direct proof; ii) an indirect proof and iii) proof by contradiction, for the following statement. "If n is an odd integer, then n + 11 is an even integer". **(07 Marks)**

3 a. Using Venn diagram, prove that, for any three sets A, B, C.
$A \Delta (B \Delta C) = (A \Delta B) \Delta C$. **(06 Marks)**

b. A survey of 500 television viewers of a sports channel produced the following information: 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, and 50 do not watch any of the three kinds of games.
i) How many viewers in the survey watch all three kinds of games?
ii) How many viewers watch exactly one of the sports? **(07 Marks)**

c. i) Find the number of arrangements of all the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's?
ii) In how many ways can one distribute eight identical balls into four distinct containers, so that the fourth container gets an odd number of balls? **(07 Marks)**

4 a. State the induction principle. By mathematical induction, prove that $(n!) \geq 2^{n-1}$ for all integers $n \geq 1$. **(06 Marks)**

b. State the Fibonacci numbers. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers, prove that

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$$ for all positive integers n. **(07 Marks)**

c. Find an explicit definition of the sequence defined recursively by
$a_n = 2a_{n-1} + 7$ for $n \geq 2$, $a_1 = 7$. **(07 Marks)**

5 a. Define Cartesian product. For any non-empty sets A, B, C prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(05 Marks)**

b. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if 'a is a multiple of b'. Find R, M(R) and draw the diagraph of R. **(05 Marks)**

c. Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1)\ R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Verify that R is an equivalence relation on $A \times A$.

ii) Determine the equivalence class $[(2, 4)]$. **(05 Marks)**

d. Let $A = \{1, 2, 3, 4, 6, 12\}$ on A, define the relation R by aRb if and only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **(05 Marks)**

6 a. State Piegeon-hole principle. ABC is an equilateral triangle whose sides are length 1cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than 1/2 cm. **(05 Marks)**

b. Let f, g, h be functions from Z to Z defined by $f(x) = x - 1$, $g(x) = 3x$,

$$h(x) = \begin{cases} 0, & \text{If } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$$

verify that $f_o(g_o h) = (f_o g)_o h$. **(05 Marks)**

c. Let $A = \{x/x \text{ is real and } x \geq -1\}$ and $B = \{x/x \text{ is real and } x \geq 0\}$ consider $f : A \to B$ defined by $f(a) = \sqrt{a+1}$, for all $a \in A$, show that f is invertible and determine f^{-1}. **(05 Marks)**

d. Let $A = \{1, 2, 3, 4\}$ and $B\{1, 2, 3, 4, 5, 6\}$. Find:

i) The number of functions possible from A to B.

ii) The number of one-to-one functions possible from A to B.

iii) The number of on-to functions from A to B.

iv) The number of on-to functions from B to A. **(05 Marks)**

7 a. Define: i) Regular graph; ii) Multi graph; iii) Complete graph. Give one example for each. **(06 Marks)**

b. Define graph isomorphism. Show that the following two graphs Fig.Q.7(b) are isomorphic. **(07 Marks)**

Fig.Q.7(b)

c. Define Euler circuit. Discuss Konigsberg bridge problem. **(07 Marks)**

8 a. Define: i) Hamilton cycle; ii) Hamilton path; iii) Planar graph. Give one example for each. **(03 Marks)**

b. Define chromatic number. Find the chromatic number of the following graphs Fig.Q.8(b). **(07 Marks)**

(i) (ii)

Fig.Q.8(b)

c. Define: i) tree; ii) rooted tree; iii) directed graph; iv) full binary tree. Give one example for each. **(07 Marks)**

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