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10MCA12

First Semester MCA Degree Examination, Dec.2015/Jan.2016

Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Using laws of set theory simplify:
$(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B)$. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18}
i) How many subsets of A contain six elements?
ii) How many six element subsets of A contain four even integers and two odd integers?
iii) How many subsets of A contain only odd integers? **(07 Marks)**

c. In a survey of 120 passengers an airline found that 48 enjoyed wine with their meals, 78 enjoyed mixed drinks and 66 enjoyed iced tea. In addition, 36 enjoyed any given pair of these beverages and 24 passengers enjoyed them all. If two passengers are selected at random from the survey sample of 120, what is the probability that,
(i) They both want only iced tea with their meals?
(ii) They both enjoy exactly two of the three beverage offerings. **(07 Marks)**

2 a. Verify that $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(06 Marks)**

b. Verify the principle of duality for the following logical equivalence:
$[\neg(p \wedge q) \to \neg p \vee (\neg p \vee q)] \Leftrightarrow (\neg p \vee q)$. **(07 Marks)**

c. Establish the validity of the following argument:

$$\begin{array}{l} p \to (q \to r) \\ p \vee \neg s \\ q \\ \hline \therefore s \to r \end{array}$$

(07 Marks)

3 a. Let p(x), q(x) denote the following open statements:
$p(x): x \le 3$, $q(x): x + 1$ is odd.
If the universe consists of all integers, what are the truth values of the following statements:
(i) $q(1)$ (ii) $\neg p(3)$ (iii) $p(7) \vee q(7)$
(iv) $p(3) \wedge q(4)$ (v) $\neg[p(-4) \vee q(-3)]$ (vi) $\neg p(-4) \wedge q(-3)$ **(06 Marks)**

b. Find whether the following argument is valid:
If a triangle has two equal sides, then it is isosceles.
If a triangle is isosceles, then it has two equal angles.
The triangle ABC does not have two equal angles.
$\therefore$ ABC does not have two equal sides. **(07 Marks)**

c. Prove that, for all real numbers x and y, if $x + y \ge 100$ then $x \ge 50$ or $y \ge 50$. **(07 Marks)**

4 a. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

b. Prove that for any positive integer n, $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$. **(07 Marks)**

c. Define the integer sequence q_0, q_1, q_2,...... recursively by a0 = 1, a1 = 1, a2 = 1 and $a_n = a_{n-2} + a_{n-3} \; \forall \; n \ge 3$, prove that
$a_{n+2} \ge (\sqrt{2})^n \; \forall \; n \ge 0$ **(07 Marks)**

5 a. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$. Determine
(i) $(A \times B)$ (ii) Number of relations from A to B
(iii) Number of binary relations on A (iv) Number of relations from A to B that contain (1, 2) and (1, 5) **(06 Marks)**

b. The functions $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined by $f(x) = 3x + 7 \ \forall x \in R$ and $g(x) = x(x^3 - 1) \ \forall x \in R$. Verify that f and g are one-to-one or not. **(07 Marks)**

c. Evaluate s(5, 4) and s(8, 6). **(07 Marks)**

6 a. Prove that any subset of size 6 from the set $s = \{1, 2, 3, ...9\}$ must contain two elements whose sum is 10. **(06 Marks)**

b. Prove that a function $f : A \rightarrow B$ is invertible if and only if it is one-to-one and onto. **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 6, 8, 12\}$ on A, define the partial ordering relation R by xRy if and only if x/y. Draw the Hasze diagram for R. **(07 Marks)**

7 a. Let G be the set of real numbers not equal to -1 and * be defined by a*b = a + b +ab. Prove that (G, *) is an abelian group. **(06 Marks)**

b. State and prove Lagrange's theorem. **(07 Marks)**

c. For a group G, prove that the function f : G → G define by $f(a) = a^{-1}$ is an isomorphism if and only if G is abelian. **(07 Marks)**

8 a. Prove that set z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a commutative ring with unity. **(06 Marks)**

b. Prove that S is a subring of R if and only if, (i) $\forall$ a, b $\in S$, we have $a + b \in S$ and $ab \in S$. **(07 Marks)**

c. Show that Z_5 is an integral domain. **(07 Marks)**

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