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10MCA12

First Semester MCA Degree Examination, June/July 2016
Discrete Mathematics

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}. A = {x/x is positive integer and $x^2 \leq 16$}, B = {1, 4}. Compute : i) $\overline{(A \cap B)}$ ii) $\overline{(A \cup B)}$ iii) $\overline{A}$ iv) $\overline{B}$. **(07 Marks)**

b. Out of 880 boys in a college, 224 played Cricket, 240 played Hockey and 336 played Basket Ball. Of the total 64 played both Basket Ball and Hockey, 80 played Cricket and Basket Ball and 40 played Cricket and Hockey. 24 boys played all the three games. **(07 Marks)**

c. i) Define Axioms of probability

ii) A problem is given to 3 students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. **(06 Marks)**

2 a. Define Tautology. Verify that $[(p \rightarrow r) \wedge (q \rightarrow r) \rightarrow [(p \vee q) \rightarrow$ are tautology. **(08 Marks)**

b. Prove the following by using the laws of logic :

i) $P \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

ii) $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. **(06 Marks)**

c. Test whether the following argument is valid if I study, I will not fail in the examination. If I do note watch TV in the evenings, I will study. I failed in the examination. Therefore I must have watched TV in the evenings. **(06 Marks)**

3 a. Define an open statement. Write the negation of "All integers are rational numbers and some rational numbers are not integers. **(07 Marks)**

b. Write down the following statements in symbolic form using quantifiers :

i) Every real number has an additive inverse

ii) The set of real numbers has multiplicative identity

iii) The integer 58 is equal to the sum of two perfect squares. **(07 Marks)**

c. Give : i) a direct proof ii) an indirect proof iii) proof by contradiction, for the following statement : "If n is an odd integer ; then (n + 9) is an even integer". **(06 Marks)**

4 a. Prove by mathematical induction method : $1 + 2 + 3 + 4 + \text{-----} + n = \frac{1}{2}n(n+1)$. **(07 Marks)**

b. A sequence $\{a_n\} = a_{n-1} + n$ for $n \geq 2$. Find a_n by explicit form. **(07 Marks)**

c. If F_i's are Fibonacci : numbers and L_i's are Lucas numbers, prove that $L_{n+4} - L_n = 5\,F_{n+2}$ for all integers $n \geq 0$. **(06 Marks)**

5 a. Let A = {a, b, c, d, e, f} and 'R' defined as follows R = {(a, b) (a, c) (b, b) (b, d) (b, c) (c, d) (d, c) (d, c) (e, f)}. Find $M(R^\infty)$ and diagraph. **(07 Marks)**

b. Let A = {1, 2, 3, 4, 5}. Define a relation R on (A × A) by $(x_1, y_1)\,R(x_2, y_2)$ off $x_1 + y_1 = x_2 + y_2$

i) Verify that R is an equivalence relation on (A × A)

ii) Determine the equivalence class of [(1, 3) (2, 4) (1, 1)]. **(07 Marks)**

c. Define the Cartesian product of two sets. For any no – empty sets A, B and C prove the following results :

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

6 a. Draw the Hasse diagram representing the positive divisors of 36. (07 Marks)

b. Find the number of ways of distributing 6 objects among 4 identical container with some container's possibility empty. (07 Marks)

c. State pigeonhole principal. Find the least number of ways of choosing 3 different numbers from 1 to 10 so that all choice have the same sum. (06 Marks)

7 a. Let G be the set of all non–zero real numbers and let a*b = ab/2, show that (G, *) is an abelian group. (07 Marks)

b. Prove that every subgroup of a cyclic group is cyclic. (07 Marks)

The parity – check matrix for an encoding function $E: Z_2^3 \to Z_2^6$ is given by:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine the associated generator matrix

ii) Does the code correct all single errors in transmission? (06 Marks)

8 a. Let $E: Z_2^m \to Z_2^n$, $m < n$ be the encoding function given by a generator matrix G or the associated parity–check matrix H, then prove that $C = E(Z_2^m)$ is a group code. (10 Marks)

b. Prove that the set Z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a commutative ring with unity. Is this ring an integral domain a field? (10 Marks)

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