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First Semester MCA Degree Examination, Dec.2016/Jan.2017
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If A, B, C are finite sets then prove the following :
i) $A\cap(B\cup C)=(A\cap B)\cup(A\cup C)$
ii) $A\cap(B\cap C)=(A\cup B)\cap(A\cup C)$ **(08 Marks)**

b. For any two sets A and B, prove the following
i) $\overline{(A\cup B)}=\overline{A}\cap\overline{B}$
ii) $\overline{(A\cap B)}=\overline{A}\cup\overline{B}$ **(06 Marks)**

c. In a survey of 260 college students the following data were obtained, 64 had taken mathematics course, 94 had taken computer science, 58 had taken business curse, 28 taken both mathematics and a business course, 26 had taken both a mathematics and computer science course, 22 had taken both computer science and business course and 14 had taken all types of courses.
i) How many of these students had taken none of the three courses?
ii) How many had taken only computer science course? **(06 Marks)**

2 a. Prove that
$[(\neg p\vee q)\wedge(p\wedge(q\wedge q))]\Leftrightarrow p\wedge q$ **(08 Marks)**

b. Define tautology. Show that
$[(p\wedge q)\wedge(p\rightarrow r)\wedge(q\rightarrow r)]\rightarrow r$ is a tautology by constructing truth table. **(06 Marks)**

c. Simplify the compound statement
$\neg[\neg((p\vee q)\wedge r)\vee\neg q]$Using laws of logic. **(06 Marks)**

3 a. Consider the following argument 9 will get grade A in this course or 9 will not graduate. If 9 do not graduate, 9 will join army 9 got grade A. therefore 9 will not join the army. Is this valid argument? Prove using rules of inferences. **(06 Marks)**

b. For any three propositions p, q, r prove that
$[(p\vee q)\rightarrow r]\Leftrightarrow[(p\rightarrow r)\wedge(q\rightarrow r)]$ **(07 Marks)**

c. Given:
i) A direct proof
ii) An indirect proof and
iii) Proof by construction, for the following statement. "If n is an odd integer, then (n+9) is an integer". **(07 Marks)**

4 a. Prove by mathematical induction that $1^2+3^2+5^2+\dots\dots\dots+(2n-1)^2=\dfrac{n(2n+1)(2n-1)}{3}$ **(06 Marks)**

b. Solve the linear recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$, $a_1 = 1$, $a_2 = 2$. **(07 Marks)**

c. Show that if seven colours used to paints 50 bicycles, at least eight bicycles must have the same colours. **(07 Marks)**

5 a. Let A = {1, 2, 3, 4, 6} and R be the relation on A defined by a R b if and only if 'a' is a multiple of 'b'. Write down the relation matrix M(R) and draw its graph. **(06 Marks)**

b. Let A {1, 2, 3, 4, 5}, define the reduction on A×A by (x, y) R (x_2, y_2) : f and only if $x_1+y_1 = x_2 + y_2$
Verify that R is equivalence relation on A×A
Determine by equivalence classes [(1, 3) (2, 4), (1, 1)] **(07 Marks)**

c. Define a poset, consider the Hasse diagram of a poset (A, R) given below in Fig Q5(c). if B = {c, d, e},
find : i) all the upper bounds of B ii) all the lower bounds of B
iii) least upper bounds of B iv) the greatest lower bound of B **(07 Marks)**

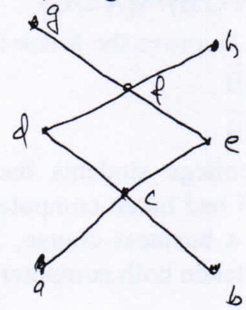

Fig. Q5(c)

6 a. Define the following :
i) Function A to B ii) One to one function iii) on to function iv) Bijective. **(08 Marks)**

b. Let A = B = R, Let f: A→B, $f(x) = 3x^4 - 1$ and g: B→A, $g(y) = \sqrt[4]{\frac{4}{3}+\frac{1}{3}}$. show that f is a bijective between A and B and g is bijective between B andA. **(06 Marks)**

c. Let f(x) = x + 2, g(x) = x − 2, & h(x) = 3x, x ←R
Find fog, gof, hog, foh, fof, fohog. **(06 Marks)**

7 a. Define the following with one example each
i) semi group ii) group iii) subgroup of a group iv) An abelian group. **(08 Marks)**

b. In a group (G, *) prove that
i) $(a^{-1})^{-1} = a$ ii) $(a*b)^{-1} = b^{-1} * a^{-1+}$ **(06 Marks)**

c. Show that a group (G, *) is an abelian group if and only if $(a \times b)^2 = a^2 * b^2$. **(06 Marks)**

8 a. Let x, y and z be elements of B^n them
i) h(n, y) = h(y, x) ii) h(x, y) ≥ 0 iii) h(x, y) = 0 iff n = y iv) h(x, y) ≤ h(x, z) + h(z, y). **(08 Marks)**

b. Let m = 2, n = 5 and $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Determine the group code $e_H ! B^2 \to B^5$ **(06 Marks)**

c. Let d be (4, 3) decoding function. Determine d(y) for the word $y_{\in} B^4$
i) y = 0110 ii) y = 1011 **(06 Marks)**

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