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On the Zagreb indices of the line graphs of the subdivision graphs

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Dedicated to Professor H. M. Srivastava on the Occasion of his Seventieth Birth Anniversary

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ABSTRACT

The aim of this paper is to investigate the Zagreb indices of the line graphs of the tadpole graphs, wheel graphs and ladder graphs using the subdivision concepts.

1. Introduction

Recently, there has been some interest in subdivision associated with Zagreb indices [6]. The fact that many interesting graphs are composed of simpler graphs that serve as their basic building blocks prompted interest in the type of relationship between the Zagreb index of a composite graph and Zagreb index of its building blocks. We refer the reader to [10] for the proof of this fact and for more information on Zagreb indices. Obviously, the Zagreb indices can be viewed as contributions of pairs of adjacent vertices to the vertex-weighted Wiener number, [1]. Curiously enough, it turns out that similar contributions of non-adjacent pairs of vertices must be taken into account when computing Zagreb co-indices of the subdivision graphs. We investigated the Zagreb indices of the p subdivision graphs of the *tadpole graphs* and *wheel graphs*, [6].

Our paper is concerned with Zagreb indices, a pair of topological indices denoted by $M_1(G)$ and $M_2(G)$ and introduced about 30 years ago, [5]. Now we recall some definitions. The first Zagreb index $M_1(G) = \sum \left[d(u)^2\right]$, where u is in the vertex set of G. The second Zagreb index is $M_2(G) = \sum[d(u)d(v)]$, where uv is an edge. The first Zagreb co-index of a graph G is defined by $\overline{M_1(G)} = \sum[d(u) + d(v)]$, where (u, v) does not belong to the edge set. The second Zagreb co-index of a graph G is defined by $\overline{M_2(G)} = \sum[d(u)d(v)]$, where (u, v) not belong to the edge set.

The *subdivision graph* [7–9] $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2, or equivalently, by inserting an additional vertex into each edge of G, [4]. The *line graph*, [2], of the graph G, written $L(G)$, is the simple graph whose vertices are the edges of G, with $ef \in E(L(G))$ when e and f have a common end point in G. The $T_{n,k}$ tadpole graph, [11], is the graph obtained by joining a cycle graph C_n to a path of length k. The ladder graph L_n is given by $L_n = K_2 \square P_n$, where P_n is a path graph. It is therefore equivalent to the *grid graph* $G_{2,n}$. The graph obtained via this definition has the advantage of looking like a ladder, having two rails and n rungs between them. Here we studied the line graph of the subdivision graph of $T_{n,k}$, L_n and W_{n+1} and calculated the *Zagreb indices* and co-indices of the graphs $L(S(T_{n,k}))$, $L(S(W_{n+1}))$ and $L(S(L_n))$. For all terminologies and notations which are not defined in this paper, refer to Harary [3].

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