USN | | | | | | | | | | 14ECS13

First Semester M.Tech. Degree Examination, June/July 2018
Probability and Random Process

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. State and prove the axioms of probability. **(06 Marks)**

b. Two balls are selected sequentially (without replacement) from an urn containing 3 red four white and 5 blue balls.
i) What is the probability that first is red and second is blue?
ii) What is the probability of selecting a white ball on the second drawing the first ball is replaced before the second is selected?
iii) What is the probability of selecting a white ball on the second draw if the first ball is not replaced before the second is selected? **(06 Marks)**

c. Explain the optical communication system to calculate the probability of error in the system. **(08 Marks)**

2 a. Define random variable. Explain the binomial random variable with an example. **(06 Marks)**

b. For the following probability mass functions find the value of the constant C:
i) $P_x(k) = C\,(0.37)^k$ K = 0, 1, 2…
ii) $P_x(k) = C\,(0.41)^k$ K = 0, 2, 4….. 12 **(06 Marks)**

c. Suppose the arrival of telephone calls at a switch can be modeled with a Poisson Pmf i.e. if X is the number of calls that arrive in t minute then $\Pr(x = k) = \frac{(\lambda t)^k}{K!} e^{-\lambda t}$ k = 0, 1, 2…, where λ is the average arrival rate in calls/min. Suppose that the average rate of calls is 10/minute:
i) What is the probability that fewer than 3 calls will be received in first 6 seconds?
ii) What is the probability that fever than 3 calls will be received in the first 6 minutes? **(08 Marks)**

3 a. Prove the following properties:
i) $F_{x/A}(-\infty) = 0 \quad F_{x/A}(\infty) = 1$
ii) $0 \le F\,x/A(x) \le 1$
iii) $F_{x/A}(x_1) \le F_{x/A}(x_2)$. **(06 Marks)**

b. Suppose a r.v has a CDF given by $F_x(x) = (1-e^{-x})\,u(x)$. Find the following quantities:
i) $P_r(x > 5)$ ii) $P_r(x < 5)$ iii) $P_r(3 < x < 7)$. **(06 Marks)**

c. A r.v has a PDF given by $f_x(x) = \frac{1}{\sqrt{8\pi}} \exp\left(-\frac{(x+3)^2}{8}\right)$. Find each of the following probability and express the answers in terms of 'Q' functions.
i) $\Pr(x \le 0)$
ii) $\Pr(x > 4)$
iii) $\Pr(|x+3| < 2)$
iv) $\Pr(|x-2| > 1)$. **(08 Marks)**

4 a. Let 'x' be a r.v. with E[x] = 1 and Var [x] = 4. Find the following:
i) E [2x-4]
ii) $E[x]^2$
iii) $E[2x-4]^2$. **(06 Marks)**

b. A random variable has an exponential PDF given by $f_x(x) = 1/b \exp(-x/b)\, u(x)$. Calculate expected value and second moment. **(06 Marks)**

c. A r.v has a uniform distribution over the interval (-a/2, a/2) for some +ve constant 'a'.
i) Find the coefficient of skewness for 'x'.
ii) Find the coefficient of Kurtosis for 'x'. **(08 Marks)**

5 a. S.T. the n^{th} moment $E\left[\left[x^n\right] = \frac{1}{j^n}\frac{d^n + (w)}{dw^n}\, w = 0\right]$. **(06 Marks)**

b. An exponential R.V. has P_{df} given by $f(x) = e^{-x}\, u(x)$. Find the characteristic function and hence find mean and variance. **(06 Marks)**

c. Find the moment generating function for standard normal distribution and hence find mean and variance. **(08 Marks)**

6 a. Consider a pair of r.v. x and y that are uniformly distributed over the unit circle so that,

$$f_{xy}(x,y) = \begin{matrix} 1/\pi & x^2 + y^2 \le 1 \\ 0 & \text{otherwise} \end{matrix}.$$

S.T. the two r.v are dependent. **(10 Marks)**

b. Two r.v x and y are $\mu_x = 2$, $\mu_y = -1$, $\sigma_x = 1$, $\sigma_y = 4$, $\rho_{xy} = 1/4$. Let u = x+2y, v = 2x-y. Find the following:
i) E[u], E[v] ii) $E[u^2]$, $E[v^2]$ iii) E[uv]. **(10 Marks)**

7 a. Define the terms:
i) Random process.
ii) Auto correlation function.
iii) Wide sense stationary process.
iv) Auto covariance. **(08 Marks)**

b. A Random process is given by x(t) = Acoswt + Bsinwt where A and B are independent zero mean random variables find:
i) Mean of x(t) ; ii) ACF ; iii) Mean ergodic wss wrt A and B. **(12 Marks)**

8 a. Explain Markov process with examples. **(06 Marks)**

b. Write short notes on:
i) Computer communication n/w.
ii) Probability of blocking in a telephone exchange. **(14 Marks)**

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