First Semester M.Tech. Degree Examination, June/July 2018 Probability Statistics and Queuing Theory

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. The probability density function of a variant x is

X		0 1		2	3	4	5	6	
	p(x)	k	3k	5k	7k	9k	11k	13k	

i) Find p(x < 4), $p(x \ge 5)$, $p(3 < x \le 6)$

ii) What will be the minimum value of k so that $p(x \le 2) > 3$.

(08 Marks)

b. I can hit A target 3 times in 5 shorts, B 2 times in 5 shorts, C 3 times in 4 shorts. They fire a valley what is the probability that: i) Two shots hit ii) Atleast two shorts hit. (08 Marks)

OR

2 a. A variable x has the probability distribution

X	-3	6	9
P(X = x)	<u>1</u>	1/2	1/3

Find E(X) and E(X²) hence evaluate E(2X + 1)².

(06 Marks)

b. x is a continuous random variable with probability density function given by

$$f(x) = kx(0 \le x < 2)$$

$$= 2k(2 \le x < 4)$$

$$= -kx + 6k(4 \le x < 6)$$

Find k and mean value of x.

(06 Marks)

c. State axioms of probability and Baye's theorem.

(04 Marks)

Module-2

3 a. Derive mean and variance of normal distribution.

(06 Marks)

b. If the probability of a bad reaction from a certain injection is 0.001 determine the chance that out of 2000 individuals more than two will get a bad reaction. (06 Marks)

c. x is a normal variate with mean 30 and standard deviation 5. Find the probabilities that :

i)
$$26 \le x \le 40$$
 ii) $x \ge 45$.

(04 Marks)

OR

a. Derive mean and variance of the binomial distribution.

(06 Marks)

b. A die is cost until 6 appears. What is the probability that it must be cost more than 5 times?

(06 Marks)

c. In 256 sets of 12 tosses of a coin in how many cases one can expect 8 heads and 4 tails.

(04 Marks)

Module-3

a. Define random process and give its classification.

(04 Marks)

b. Suppose a Markov chain with three states has the probability transition matrix:

$$p = \begin{bmatrix} 0 & 0 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}$$
. Determine whether or not this Markov chain is irreducible. (04 Marks)

c. Derive the mean and variance of a Poisson process.

(08 Marks)

16/17SCS/SCN/SCE/SSE/SFC/SIT/LNI14

OR

6 a. The transition probability matrix of a Markov chain have 3 states 1, 2, 3 is,

$$p = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

The initial distribution is $p^{(0)} = [0.7 \ 0.2 \ 0.1]$. Find the probability of $p[X_2 = 3]$. Find the probability of $p[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. (04 Marks)

b. Prove that the difference of two independent Poisson process is not Poisson. (06 Marks)

c. Define Ergodicity and explain Ergodic process.

Module-4

7 a. A coin was tossed 400 times and the head turned up 216 times test the hypothesis that coin is unbiased at 5% level of significance. (06 Marks)

b. Two samples of sizes 9 and 8 give the sum of squares of deviation from their respective means equal to 160 inches² and 91 inches² respectively can these be regarded as drawn from the same normal population? $[F_{0.05} = 3.73]$. (04 Marks)

c. Write a short note on: i) Errors in testing ii) Chi-square distribution. (06 Marks)

OR

8 a. In a city A 20% of random sample of 900 school boys had a certain slight physical defect. In another city B 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? (06 Marks)

b. Explain the procedure to test significance and goodness fit. (04 Marks)

c. Five dice were thrown 96 times and the number of times 4, 5 or 6 were thrown were,

No. of dice showing 4, 5 or 6	5	4	3	2	1>	0
Frequency	8	18	35	24	10	1

Find the probability of getting this result by chance.

(06 Marks)

(06 Marks)

Module-5

- 9 a. A Television repairman finds that the time spent on his jobs has an exponential distribution with mean 30 mins. If he repairs the sets in the order in which they come in and if the arrivals of sets are approximately Poisson with an average rate of 10 per 8 hours day which is the repairs than idle time each day? Find the expected number of units in the system and in the queue. (06 Marks)
 - b. State and explain little law.c. Explain the types of stochastic processes.

(06 Marks)

(04 Marks)

OR

- 10 a. Calls in a telephone system arrive randomly at an exchange at the rate of 140 per hour. If there is a large number of lines available to handle calls, that lasts an average of 3 mins, what is the average number of lines in use? Estimate the 90th percentile of number of lines in use.

 (04 Marks)
 - b. Explain the following:
 - i) M/M/1 queuing system
 - ii) M/M/S queuing system.

(08 Marks)

c. Briefly explain the birth-death process.

(04 Marks)

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