CBCS Scheme

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15MAT41

Fourth Semester B.E. Degree Examination, June/July 2018

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note:* 1. *Answer any FIVE full questions, choosing one full question from each module.
2. Use of statistical tables is permitted.

Module-1

1 a. Use Taylor's series method to find y at x = 1.1, considering terms upto third degree given that $\frac{dy}{dx} = x + y$ and y(1) = 0. **(05 Marks)**

b. Using Runge-Kutta method, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$; y(0) = 1, taking h = 0.2. **(05 Marks)**

c. Given $\frac{dy}{dx} = x^2 - y$, y(0) = 1 and the values y(0.1) = 0.90516, y(0.2) = 0.82127, y(0.3) = 0.74918, evaluate y(0.4), using Adams-Bashforth method. **(06 Marks)**

OR

2 a. Using Euler's modified method, find y(0.1) given $\frac{dy}{dx} = x - y^2$, y(0) = 1, taking h = 0.1. **(05 Marks)**

b. Solve $\frac{dy}{dx} = xy$; y(1) = 2, find the approximate solution at x = 1.2, using Runge-Kutta method. **(05 Marks)**

c. Solve $\frac{dy}{dx} = x - y^2$ with the following data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762, compute y at x = 0.8, using Milne's method. **(06 Marks)**

Module-2

3 a. Using Runge-Kutta method of order four, solve $y'' = y + xy'$, y(0) = 1, y′(0) = 0 to find y(0.2). **(05 Marks)**

b. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(05 Marks)**

c. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x) J_n(\beta x) dx = 0$, if α ≠ β. **(06 Marks)**

OR

4 a. Given $y'' = 1 + y'$; y(0) = 1, y′(0) = 1, compute y(0.4) for the following data, using Milne's predictor-corrector method.

y(0.1) = 1.1103	y(0.2) = 1.2427	y(0.3) = 1.399
y′(0.1) = 1.2103	y′(0.2) = 1.4427	y′(0.3) = 1.699.

(05 Marks)

b. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(05 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$. **(06 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equations in polar form. **(05 Marks)**

b. Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$, using Cauchy's residue theorem. **(05 Marks)**

c. Find the bilinear transformation which maps $z = \infty, i, 0$ on to $w = 0, i, \infty$. **(06 Marks)**

OR

6 a. State and prove Cauchy's integral formula. **(05 Marks)**

b. If $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$, find the corresponding analytic function $f(z) = u + iv$. **(05 Marks)**

c. Discuss the transformation $w = z^2$. **(06 Marks)**

Module-4

7 a. Derive mean and standard deviation of the binomial distribution. **(05 Marks)**

b. If the probability that an individual will suffer a bad reaction from an injection of a given serum is 0.001, determine the probability that out of 2000 individual (i) exactly 3 (ii) more than 2 individuals will suffer a bad reaction. **(05 Marks)**

c. The joint probability distribution for two random variables X and Y is as follows:

X \ Y	-3	-2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Determine: i) Marginal distribution of X and Y ii) Covariance of X and Y iii) Correlation of X and Y **(06 Marks)**

OR

8 a. Derive mean and standard deviation of exponential distribution. **(05 Marks)**

b. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. Given $P(0 < z < 1.2263) = 0.39$ and $P(0 < z < 1.14757) = 0.43$. **(05 Marks)**

c. The joint probability distribution of two random variables X and Y is as follows:

Y \ X	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute: i) E(X) and E(Y) ii) E(XY) iii) COV(X, Y) iv) $\rho(X, Y)$ **(06 Marks)**

Module-5

9 a. Explain the terms: i) Null hypothesis ii) Type I and Type II errors. **(05 Marks)**

b. The nine items of a sample have the values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? **(05 Marks)**

c. Given the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ then show that A is a regular stochastic matrix. **(06 Marks)**

OR

10 a. A die was thrown 9000 times and of these 3220 yielded a 3 or 4, can the die be regarded as unbiased? **(05 Marks)**

b. Explain: i) Transient state ii) Absorbing state iii) Recurrent state **(05 Marks)**

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run, how often does he study? **(06 Marks)**