CBCS SCHEME

USN | | | | | | | | | | | **15MAT31**

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics - III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. An alternating current after passing through a rectifier has the form,

$$I = \begin{cases} I_0 \sin x & \text{for } 0 < x < \pi \\ 0 & \text{for } \pi < x < 2\pi \end{cases}$$

where I_0 is the maximum current and the period is 2π. Express I as a Fourier series. **(08 Marks)**

b. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data: **(08 Marks)**

x^0	0	45	90	135	180	225	270	315
y	2	1.5	1	0.5	0	0.5	1	1.5

OR

2 a. Obtain the Fourier series expansion of the function, $f(x) = |x|$ in $(-\pi, \pi)$ and hence deduce that,

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$$ **(06 Marks)**

b. Find the Fourier series expansion of the function,

$$f(x) = \begin{cases} \pi x & \text{in } 0 \le x \le 1 \\ \pi(2 - x) & \text{in } 1 \le x \le 2 \end{cases}$$ **(05 Marks)**

c. The following table gives the variations of periodic current over a period.

t(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A(amplitude)	1.98	1.30	1.05	1.3	-0.88	-0.25	1.98

Show by harmonic analysis that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of first harmonic. **(05 Marks)**

Module-2

3 a. Find the complex Fourier transform of the function $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$. Hence evaluate

$$\int_0^\infty \frac{\sin x}{x} dx.$$ **(06 Marks)**

b. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. **(05 Marks)**

c. Compute the inverse z-transforms of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

4 a. Find the z-transform of $e^{-an}n + \sin n\frac{\pi}{4}$. **(06 Marks)**

b. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using z-transform. **(05 Marks)**

c. Find the Fourier cosine transform of, $f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4 - x & 1 < x < 4 \\ 0 & x > 4 \end{cases}$. **(05 Marks)**

Module-3

5 a. Find the Correlation coefficient and equations of regression lines for the following data:

x	1	2	3	4	5
y	2	5	3	8	7

(06 Marks)

b. Fit a straight line to the following data:

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

(05 Marks)

c. Find a real root of the equation $xe^x = \cos x$ correct to three decimal places that lies between 0.5 and 0.6 using Regula-falsi method. **(05 Marks)**

OR

6 a. The following regression equations were obtained from a correlation table.

$y = 0.516x + 33.73$

$x = 0.516y + 32.52$

Find the value of (i) Correlation coefficient (ii) Mean of x's (iii) Mean of y's. **(06 Marks)**

b. Fit a second degree parabola to the following data:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(05 Marks)

c. Use Newton-Raphson's method to find a real root of $x\sin x + \cos x = 0$ near $x = \pi$, carry out three iterations. **(05 Marks)**

Module-4

7 a. The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in °C and P is the percentage of lead in the alloy:

P%	60	70	80	90
t	226	250	276	304

Find the melting point of the alloy containing 84% of lead, using Newton's interpolation formula. **(06 Marks)**

b. Apply Lagrange's interpolation formula to find a polynomial which passes through the points $(0, -20)$, $(1, -12)$, $(3, -20)$ and $(4, -24)$ **(05 Marks)**

c. Find the approximate value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by Simpson's $\frac{3}{8}^{th}$ rule by dividing it into 6 equal parts. **(05 Marks)**

OR

8 a. From the following table :

x°	10	20	30	40	50	60
cos x	0.9848	0.9397	0.8660	0.7660	0.6428	0.5

Calculate $\cos 25^\circ$ using Newton's forward interpolation formula. **(06 Marks)**

b. Use Newton's divided difference formula and find f(6) from the following data:

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

(05 Marks)

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Weddle's rule by taking equidistant ordinates. **(05 Marks)**

Module-5

9 a. Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ with the help of Green's theorem in a plane. **(06 Marks)**

b. Solve the variational problem $\delta\int_0^1 (12xy + y'^2)dx = 0$ under the conditions y(0) = 3, y(1) = 6. **(05 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them. **(05 Marks)**

OR

10 a. A cable hangs freely under gravity from the fixed points. Show that the shape of the curve is a catenary. **(06 Marks)**

b. Use Stoke's theorem to evaluate for $\vec{F} = (x^2 + y^2)i - 2xyj$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. **(05 Marks)**

c. Evaluate $\iint_S (yzi + zxj + xyk).\hat{n}ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. **(05 Marks)**

* * * * *