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MATDIP301

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Advanced Mathematics - I

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions.*

1   a.   Express the $\dfrac{3}{1+i} - \dfrac{1}{2-i} + \dfrac{1}{1-i}$ in the form of a + ib.   **(06 Marks)**

   b.   Find the cube roots of 1 - i.   **(07 Marks)**

   c.   Prove that $\left(\dfrac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^{n} = \cos n\theta + i\sin n\theta$.   **(07 Marks)**

2   a.   Find the nth derivative of $e^{ax}\cos(bx + c)$.   **(07 Marks)**

   b.   Find the nth derivative of $\dfrac{x}{(x-1)(2x+3)}$.   **(06 Marks)**

   c.   If $y = a\cos(\log x) + b\sin(\log x)$ prove that $x^{2}y_{n+2} + (2n+1)xy_{n+1} + (n^{2}+1)y_{n} = 0$.   **(07 Marks)**

3   a.   With usual notations P.T $\tan\phi = \dfrac{r d\theta}{dr}$.   **(06 Marks)**

   b.   Find the angle between the pairs of curves

   $r = a\log\theta \qquad r = \dfrac{a}{\log\theta}$.   **(07 Marks)**

   c.   Find the Pedal equation to the curve $r = a(1+\sin\theta)$.   **(07 Marks)**

4   a.   State and prove Euler's theorem of Homogeneous functions.   **(06 Marks)**

   b.   If $u = f(x-y, y-z, z-x)$

   P.T $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.   **(07 Marks)**

   c.   If $u = \tan^{-1}x + \tan^{-1}y$, $V = \dfrac{x+y}{1-xy}$

   S.T $\dfrac{\partial(u,v)}{\partial(x,y)} = 0$.   **(07 Marks)**

5   a.   Obtain the Reduction formula for $\int \sin^{m}x \cos^{n}x\, dx$. Where m, n are positive integers.   **(07 Marks)**

   b.   Evaluate $\displaystyle\int_{1}^{2}\int_{0}^{2-y} xy\, dx\, dy$.   **(06 Marks)**

   c.   Evaluate $\displaystyle\int_{0}^{3}\int_{0}^{2}\int_{0}^{1} (x+y+z)\, dz\, dx\, dy$.   **(07 Marks)**

6   a.   Prove that $\left|\underline{\left(\dfrac{1}{2}\right)} = \sqrt{\pi}\right.$.   **(06 Marks)**

   b.   Prove that $\displaystyle\int_{0}^{\infty} x^{2}e^{-x^{4}}dx \times \int_{0}^{\infty} e^{-x^{4}}dx = \dfrac{\pi}{8\sqrt{2}}$.   **(07 Marks)**

c. Evaluate the Integral $\int_0^1 x^5(1-x)^6 \, dx$. **(07 Marks)**

7 a. Solve $(D^3 - 3D - 2)y = 0$. **(06 Marks)**
   b. Solve $(y'' + y) = e^{-x} + \cos x + x^3$. **(07 Marks)**
   c. Solve $y'' - 2y' + y = xe^x \sin x$. **(07 Marks)**

8 a. Solve $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{\sin y + y\cos y}$. **(06 Marks)**

   b. Solve $x \log x \dfrac{dy}{dx} + y = 2 \log x$. **(07 Marks)**

   c. Solve $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy = 0$. **(07 Marks)**

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