### ACHARYA INSTITUTE OF TECHNOLOGY Bangalore - 560090

# CRGS Scheme

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## Second Semester B.E. Degree Examination, Dec.2016/Jan.2017 **Engineering Mathematics - II**

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

a. Solve  $(D-2)^2$   $y = 8(e^{2x} + x + x^2)$  by inverse differential operator method. b. Solve  $(D^2 - 4D + 3)$   $y = e^x \cos 2x$ , by inverse differential operator method. (06 Marks)

(05 Marks)

c. Solve by the method of variation of parameters  $y'' - 6y' + 9y = \frac{e^{3x}}{c^2}$ . (05 Marks)

a. Solve (D² - 1)y = x sin 3x by inverse differential operator method.
b. Solve (D³ - 6D² + 11D - 6)y = e²x by inverse differential operator method.
c. Solve (D² + 2D + 4) y = 2x² + 3 e⁻x by the method of undetermined coefficient. (06 Marks)

(05 Marks)

(05 Marks)

a. Solve  $x^3y''' + 3x^2y'' + xy' + 8y = 65\cos(\log x)$ . (06 Marks) b. Solve  $xy p^2 + p(3x^2 - 2y^2) - 6xy = 0$ . (05 Marks) c. Solve the equation  $y^2(y - xp) = x^4 p^2$  by reducing into Clairaut's form, taking the substitution

 $x = \frac{1}{y}$  and  $y = \frac{1}{y}$ . (05 Marks)

a. Solve  $(2x + 3)^2$  y" - (2x + 3) y' - 12y = 6x. b. Solve  $p^2 + 4x^5p - 12x^4y = 0$ . c. Solve  $p^3 - 4xy$  p  $+ 8y^2 = 0$ . (06 Marks)

(05 Marks)

(05 Marks)

Module-3

a. Obtain the partial differential equation by eliminating the arbitrary function. Z = f(x + at) + g(x - at).(06 Marks)

b. Solve  $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ , for which  $\frac{\partial z}{\partial y} = -2 \sin y$ , when x = 0 and  $z \neq 0$ , when y is an odd

multiple of  $\pi/2$ . (05 Marks)

c. Find the solution of the wave equation  $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$  by the method of separation of variables. (05 Marks)

OR

a. Obtain the partial differential equation by eliminating the arbitrary function  $\ell x + my + nz = \phi(x^2 + y^2 + z^2).$ (06 Marks)

b. Solve  $\frac{\partial^2 z}{\partial v^2} = z$ , given that, when y = 0,  $z = e^x$  and  $\frac{\partial z}{\partial v} = e^{-x}$ . (05 Marks)

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c. Derive one dimensional heat equation 
$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$
. (05 Marks)

### Module-4

7 a. Evaluate 
$$\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z) \, dy \, dx \, dz$$
. (06 Marks)

b. Evaluate 
$$\int_{0}^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \, dy \, dx$$
 by changing the order of integration. (05 Marks)

c. Evaluate 
$$\int_{0}^{4} x^{\frac{3}{2}} (4-x)^{5/2} dx$$
 by using Beta and Gamma function. (05 Marks)

8 a. Evaluate 
$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$$
 by changing to polar co-ordinates. Hence show that 
$$\int_0^\infty e^{-x^2} dx = \sqrt{\pi/2}.$$
 (06 Marks)

- b. Find by double integration, the area lying inside the circle  $r = a \sin \theta$  and outside the cardioid (05 Marks)
- Obtain the relation between beta and gamma function in the form

$$\beta (m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$
 (05 Marks)

9 a. Find i) 
$$L\{e^{-3t} (2 \cos 5 t - 3 \sin 5 t)\}$$
 ii)  $L\{\frac{\cos at - \cos bt}{t}\}$ . (06 Marks)

b. If a periodic function of period 2a is defined by 
$$f(t) = \begin{cases} t & \text{if } 0 \le t \le a \\ 2a - t & \text{if } a \le t \le 2a \end{cases} \text{ then show that } L\{f(t)\} = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right). \tag{05 Marks}$$

c. Solve the equation by Laplace transform method. y''' + 2y'' - y' - 2y = 0. Given y(0) = y'(0) = 0, y''(0) = 6. (05 Marks)

#### OR

10 a. Find L<sup>-1</sup> 
$$\left\{ \frac{s+3}{s^2-4s+13} \right\}$$
. (06 Marks)

b. Find 
$$L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$$
 by using Convolution theorem. (05 Marks)

$$c. \quad \text{Express } f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \sin 2t, & \pi \le t < 2\pi \\ \sin 3t, & t \ge 2\pi \end{cases} \qquad \text{in terms of unit step function and hence find its}$$