15MAT21

Second Semester B.E. Degree Examination, Dec.2018/Jan.2019 Engineering Mathematics - II

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

1 a. Solve
$$\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$$
 by inverse differential operator method. (06 Marks)

b. Solve
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = e^x \cos x$$
 by inverse differential operator method. (05 Marks)

c. Solve
$$(D^2 + 1)y = \csc x$$
 by the method of variation of parameters. (05 Marks)

2 a. Solve
$$(D^3 - 5D^2 + 8D - 4)y = (e^x + 1)^2$$
 by inverse differential operator method. (06 Marks)

b. Solve
$$\frac{d^2y}{dx^2} - y = (1 + x^2)e^x$$
 by inverse differential operator method. (05 Marks)

c. Solve
$$(D^2 - 3D + 2)y = x^2 + e^{3x}$$
 by the method of undetermined coefficients. (05 Marks)

3 a. Solve
$$x^2y'' + xy' + y = \sin^2(\log x)$$

b. Solve $p^2 + p(x + y) + xy = 0$ (06 Marks)

c. Solve
$$p + p(x + y) + xy = 0$$
 (05 Marks)
c. Solve $p = \sin(y - xp)$. Also find its singular solution. (05 Marks)

4 a. Solve
$$(1 + 2x)^2 y'' - 6(1 + 2x)y' + 16y = 8(1 + 2x)^2$$
 (06 Marks)

b. Solve
$$xp^2 - 2yp + x = 0$$
 (05 Marks)

c. Solve
$$y = 2px + y^2p^3$$
 (05 Marks)

Module-3

a. Form the partial differential equation from z = f(x + ay) + g(x - ay) by eliminating arbitrary functions f and g.

b. Solve
$$\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$$
, given $\frac{\partial z}{\partial y} = -2 \cos y$ when $x = 0$ and when y is odd multiple of π z = 0. (05 Marks)

c. Derive one dimensional wave equation
$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$
. (05 Marks)

OR

a. Obtain the partial differential equation by eliminating a, b, c from $z = ax^2 + bxy + cy^2$. (06 Marks)

b. Solve
$$\frac{\partial^2 z}{\partial y^2} = z$$
, given that $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$ when $y = 0$. (05 Marks)

Obtain the various possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of variables separable. (05 Marks)

7 a. Evaluate
$$\int_{1}^{3} \int_{1/x}^{1} \int_{0}^{\sqrt{xy}} xyz \, dz \, dy \, dx$$

(06 Marks)

(06 Marks)

- b. Change the order of integration in $\int_{0}^{\infty} \frac{x dx dy}{x^2 + y^2}$ and hence evaluate. (05 Marks)
- c. Prove that $\int_{0}^{\pi/2} \sqrt{\sin \theta} \, d\theta \times \int_{0}^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$ a. Evaluate $\int_{0}^{a} \int_{0}^{\sqrt{a^2 x^2}} y^2 \sqrt{x^2 + y^2} \, dy dx$ by changing into polar coordinates. (05 Marks)

(06 Marks)

Find by double integration the area bounded between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (05 Marks)

c. Prove that $\beta(m,n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$ (05 Marks)

a. Find (i) L{ $te^{-2t} \sin^2 t$ } (ii) L $\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$ (06 Marks)

(05 Marks)

Given $f(t) = t^2$, 0 < t < 2a and f(t + 2a) = f(t), find $L\{f(t)\}$ Using Laplace transforms solve the differential equation $y'' - 2y' + y = e^{2t}$ with y(0) = 0 and y'(0) = 1. (05 Marks)

with
$$y(0) = 0$$
 and $y'(0) = 1$.

b. Using convolution theorem find $L^{-1}\left\{\frac{1}{(s)}\right\}$ (05 Marks)

interms of unit step function and hence find its Laplace transforms. (05 Marks)