

# CBCS SCHEME

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17MAT11

## First Semester B.E. Degree Examination, Dec.2018/Jan.2019 Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(1+x)(1+2x)}$ . (06 Marks)
- b. Prove that the following curves cut orthogonally  $r^n = a^n \cos n \theta$  and  $r^n = b^n \sin n \theta$ . (07 Marks)
- c. Find the radius of curvature of the curve  $r^n = a^n \cos n \theta$ . (07 Marks)

OR

- 2 a. If  $\cos^{-1}(y/b) = \log(x/n)^n$ , then show that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$ . (06 Marks)
- b. Find the pedal equation of the curve  $r^2 = a^2 \sec 2\theta$ . (07 Marks)
- c. Find the radius of curvature for the curve  $y^2 = \frac{4a^2(2a-x)}{x}$ , where the curve meets the  $x$ -axis. (07 Marks)

### Module-2

- 3 a. Obtain the Taylor's expansion of  $\log_e x$  about  $x = 1$  upto the term containing fourth degree. (06 Marks)
- b. If  $u = \operatorname{cosec}^{-1} \left( \frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}} \right)$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{6} \tan u$ . (07 Marks)
- c. If  $u = x + 3y^2 - z^3$ ,  $v = 4x^2yz$ ,  $w = 2z^2 - xy$ , find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$  at  $(1, -1, 0)$ . (07 Marks)

OR

- 4 a. Evaluate  $\lim_{x \rightarrow 0} \left\{ \frac{\sin 2x - 2 \sin x}{x^3} \right\}$ . (06 Marks)
- b. Obtain the Maclaurin's expansion of the function  $\log(1+x)$  upto 4<sup>th</sup> degree terms. (07 Marks)
- c. If  $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ . (07 Marks)

### Module-3

- 5 a. A particle moves along the curve,  $x = 1 - t^3$ ,  $y = 1 + t^2$  and  $z = 2t - 5$ . Find the components of velocity and acceleration at  $t = 1$  in the direction  $2i + j + 2k$ . (06 Marks)
- b. If  $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ , find  $a, b, c$  such that  $\operatorname{Curl} \vec{F} = \vec{0}$  and then find  $\phi$  such that  $\vec{F} = \nabla \phi$ . (07 Marks)
- c. Prove that  $\operatorname{div}(\phi \vec{A}) = \phi (\operatorname{div} \vec{A}) + \operatorname{grad} \phi \cdot \vec{A}$ . (07 Marks)

OR

- 6 a. The position vector of a particle at time  $t$  is  $\vec{r} = \cos(t-1) \mathbf{i} + \sin h(t-1) \mathbf{j} + t^3 \mathbf{k}$ . Find the velocity and acceleration at  $t = 1$ . (06 Marks)
- b. If  $\vec{F} = \nabla(xy^3z^2)$ , find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$  at the point  $(1, -1, 1)$ . (07 Marks)
- c. Prove that  $\text{Curl}(\phi \vec{A}) = \phi(\text{curl } \vec{A}) + \text{grad } \phi \times \vec{A}$ . (07 Marks)

**Module-4**

- 7 a. Find the reduction formula for  $\int_0^{\pi/2} \sin^n x \, dx$ . (06 Marks)
- b. Solve  $x^3 \frac{dy}{dx} - x^2 y = -y^4 \cos x$ . (07 Marks)
- c. Show that the family of parabolas  $y^2 = 4a(x+a)$  is self orthogonal. (07 Marks)

**OR**

- 8 a. Evaluate  $\int_0^{\infty} \frac{x^2}{(1+x^2)^{7/2}} dx$ . (06 Marks)
- b. Solve  $(y^2 e^{xy^2} + 4x^3)dx + (2xy e^{xy^2} - 3y^2)dy = 0$ . (07 Marks)
- c. A body in air at  $25^\circ\text{C}$  cools from  $100^\circ\text{C}$  to  $75^\circ\text{C}$  in 1 minute. Find the temperature of the body at the end of 3 minutes. (07 Marks)

**Module-5**

- 9 a. Find the rank of the matrix
- $$\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$
- (06 Marks)
- b. Find the numerically largest eigen value and the corresponding eigen vector of the matrix by power method :
- $$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$
- by taking the initial approximation to the eigen vector as  $[1, 0.8, -0.8]$ .  
Perform 3 iterations. (07 Marks)
- c. Show that the transformation :
- $$y_1 = 2x_1 - 2x_2 - x_3, \quad y_2 = -4x_1 + 5x_2 + 3x_3 \text{ and } y_3 = x_1 - x_2 - x_3$$
- is regular and find the inverse transformation. (07 Marks)

**OR**

- 10 a. Solve  $20x + y - 2z = 17$  ;  $3x + 20y - z = -18$  ;  $2x - 3y + 20z = 25$  by Gauss – Seidel method. (06 Marks)
- b. Diagonalize the matrix  $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ . (07 Marks)
- c. Reduce the quadratic form  $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$  into Canonical form, using orthogonal transformation. (07 Marks)

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