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10MAT11

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two full questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) If $y = a^{mx}$ then y_n is,

A) $(m\log a)^n a^{mx}$ B) $(\log a)a^{mx}$ C) $(m\log a)a^{mx}$ D) $(m\log a)^{n-1}a^{mx}$

ii) If $y = \sin 2x$ then y_n is,

A) $2^n \cos\left(2x + \frac{n\pi}{2}\right)$ B) $2^n \sin\left(2x + \frac{n\pi}{2}\right)$ C) $2^{n-1}\sin\left(2x + \frac{n\pi}{2}\right)$ D) None of these

iii) If e^x and e^{-x} in $3 \le x \le 7$ satisfies all the conditions of cauchy's mean value theorem then the value of C is,

A) 3 B) 4 C) 5 D) 6

iv) Maclaurin's series expansion of coshx is,

A) $1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \ldots$ B) $1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \frac{x^6}{\lfloor 6} + \ldots$

C) $x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \ldots$ D) $x + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \ldots$

b. If $y = \log\left[x + \sqrt{1+x^2}\right]$, prove that $(1+x^2)y_{n+2} + (2n+1)ny_{n+1} + n^2 y_n = 0$. **(06 Marks)**

c. Show that the constant C for Rolle's theorem for the function $\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in [a, b] is the geometric mean between a and b $(0 \notin [a,b])$. **(06 Marks)**

d. By using Maclaurin's series prove that

$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}.....$ **(04 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) $\underset{x\to 0}{\text{Lt}} \frac{\tan x - x}{x^2 \tan x}$ is,

A) 1/3 B) 2/3 C) 4/3 D) 5/3

ii) The angle between radius vector and the tangent of $r = ae^{\theta\cot\alpha}$ is:

A) $\frac{\pi}{2} - \theta$ B) α C) θ D) None of these

iii) The pedal equation of the curve $r = a[1 + \cos\theta]$ is,

A) $r^3 = 2ap^2$ B) $r^2 = 2ap^3$ C) $r = ap$ D) $p = \frac{r^2}{a}$

iv) The radius of curvature of the curve $y = e^x$ at the point where it crosses the y-axis is,

A) 2 B) $\sqrt{2}$ C) $2\sqrt{2}$ D) $\frac{1}{2}\sqrt{2}$

b. Evaluate $\underset{x\to\infty}{\text{Lt}} \left(\frac{ax+1}{ax-1}\right)^x$ **(06 Marks)**

c. Find the acute angles between the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$. **(06 Marks)**

d. Find the radius of curvature of the curve $r = a\sin(n\theta)$ at the pole. **(04 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $u = x^m y^n$ then the value of $\frac{\partial^2 u}{\partial y \partial x}$ is,

A) $mnx^{m-1}y^{n-1}$ B) $mx^{n-1} + ny^{n-1}$

C) $m^m x^y y^n + nx^m y^{n-1}$ D) 0

ii) A set of necessary conditions for f(x, y) to have a maximum or minimum is that:

A) $t_n = 0$ & $f_y \neq 0$ B) $f_x \neq 0$, $f_y = 0$

C) $f_x = 0$ and $f_y = 0$ D) None of these

iii) If $x = uv$ and $y = \frac{u}{v}$ then the Jacobian of $\frac{\partial(x\ y)}{\partial(u\ v)}$ is,

A) 1 B) 0 C) $\frac{-2u}{v}$ D) $\frac{2v}{u}$

iv) If an error of 1% is made in measuring its length and breadth, the percentage error in the area of a rectangle is,

A) 0.2% B) 0.02% C) 2 % D) 1%

b. If $u = \tan^{-1}\left(\frac{y}{x}\right)$, when $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$ find $\frac{du}{dt}$. **(06 Marks)**

c. If $x + y + z = u$, $y + z = uv$, $z = uvw$ show that the Jacobian of, $\frac{\partial(x\ y\ z)}{\partial(u\ v\ w)} = u^2 v$. **(06 Marks)**

d. The focal length of a mirror is given by the formula $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$. If equal errors 'e' are made in the determination of u and v. Show that the resulting relative error in f is $e\left(\frac{1}{u} + \frac{1}{v}\right)$. **(04 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\vec{R} = xi + yj + zk$ and $r = |\vec{R}|$ then ∇r^n is:

A) $nr^{n-2}\vec{R}$ B) $(n-2)r^n\vec{R}$ C) $r^n\vec{R}$ D) $(n-1)r^{n-1}\vec{R}$

ii) $\nabla \times \left(r^n \vec{r}\right) = 0$ then it is called,

A) Solenoidal B) Irrotational C) Rotational D) None of these

iii) If $\phi = x^2 + y^2 + z^2$ then $\nabla^2\phi$ is,

A) 1 B) –1 C) 2 D) 6

iv) The cylindrical polar-coordinates are $(\rho\ \phi\ z)$ given by,

A) $x = \rho\cos\phi$ $y = \rho\sin\phi$ $z = z$ B) x, y, z

C) $x = \sin\theta$ $y = \cos\theta$ D) None of these

b. If $\vec{F} = \nabla(xy^3z^2)$ find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ at the point (1, -1, 1). **(06 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$ **(06 Marks)**

d. Prove that in orthogonal curvilinear co-ordinate system cylindrical system is orthogonal. **(04 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^{\pi/8} \cos^3(4x)dx$ is,

A) 1/3 B) 1/6 C) $\pi/3$ D) 1/2

ii) The value of $\int_0^1 x^2(1-x^2)^{\frac{3}{2}}dx$ is,

A) $\pi/32$ B) 1/32 C) $\pi^2/32$ D) $\pi/16$

iii) If the axis of the revolution is the y-axis then the surface area of revolution is,

A) $\int_{x_1}^{x_2} 2\pi y ds$ B) $\int_{y_1}^{y_2} 2\pi x ds$ C) $\int_{y_1}^{y_2} 2\pi\sqrt{1+y'^2}dy$ D) $\int_{x_1}^{x_2} 2\pi\sqrt{1+\left(\frac{dx}{dy}\right)^2}dx$

iv) The area of the cardiode $r = a[1-\cos\theta]$ is,

A) $3\pi a^2/2$ B) $3\pi/2$ C) $a^2/2$ D) None of these

b. By applying differential under the integral sign. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx$ $(\alpha \geq 0)$ where α is the parameter. Hence find $\int_0^1 \frac{x^3-1}{\log x}dx$. **(06 Marks)**

c. Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$ where n is a +ve integers. **(06 Marks)**

d. Find the entire length of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. **(04 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) The solution of the differential equation, $\frac{dy}{dx} + \frac{y}{x} = x^2$ under the condition that y = 1 when x = 1 is,

A) $4xy = x^3 + 3$ B) $4xy = x^4 + 3$ C) $4xy = y^4 + 3$ D) $4xy = y^3 + 3$

ii) The integrating factor of $\frac{dy}{dx} + \frac{y}{x} = x$ is,

A) x B) x^{-1} C) y^n D) y^{-n}

iii) For the differential equation, $M(x,y)dx + N(x,y)dy = 0$ the condition for exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ is,

A) Necessary condition B) Sufficeint condition
C) Necessary and sufficient condition D) General solution.

iv) To find the orthogonal trajectories in polar co-ordinates $\frac{dr}{d\theta}$ should be replaced by,

A) $\frac{d\theta}{dr}$ B) $\frac{d\theta}{rdr}$ C) $-\frac{1}{r^2}\frac{d\theta}{dr}$ D) $-r^2\frac{d\theta}{dr}$

b. Solve $xdy - ydx = \sqrt{x^2+y^2}dx$. **(06 Marks)**

c. Solve $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x}(\log z)^2$. **(06 Marks)**

d. Find the orthogonal trajectories of the family of curves $r^n \sin n\theta = a^n$ where a is the parameter. **(04 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) The rank of the matrix $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$ is,

A) 3 B) 2 C) 1 D) 4

ii) If $m < n$ then $\rho(A) < n$ and consequently the system has:

A) Finite solution B) Infinite many nontrivial solution
C) Trivial solution D) None of these

iii) In Gauss Jordan method the co-efficient matrix can be reduced to,

A) Echelon form B) Diagonal matrix C) Triangular form D) Null matrix

iv) If $3x + 2y + z = 0$, $x + 4y + z = 0$, $2x + y + 4z = 0$ be a system of equations, then

A) it is consistent
B) it has only the trivial solution $x = 0$, $y = 0$, $z = 0$
C) it can be reduced to a single equation and so a solution does not exist
D) None of these

b. Using elementary transformation reduce the following matrices to the echelon form hence find the rank of $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$. **(06 Marks)**

c. Test for consistency and solve the system $x + y + z = 4$, $2x + y - z = 1$, $x - y + 2z = 2$. **(06 Marks)**

d. Solve the system by Gauss Jordan method,
$x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ **(04 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) If two eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15 then the 3rd eigen value is,

A) 0 B) 1 C) 2 D) 3

ii) If X is an eigen value of a non singular matrix A then eigen value of A^{-1} is,

A) $\frac{1}{\lambda}$ B) $-\frac{1}{\lambda}$ C) $\frac{1}{A}$ D) None of these

iii) The quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$ is,

A) $x^2 - 4xy + 4y^2$ B) $x^2 + 4xy - 4y^2$ C) $x^2 + 4xy$ D) None of these

iv) The index and signature of the quadratic form $x_1^2 + 2x_2^2 - 3x_3^2$ are respectively.

A) Index = 2, Signature = 1 B) Index = 1, Signature = 2
C) Index = 1, Signature = 1 D) None of these

b. Find the eigen values and eigen vectors of the matrix, $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ **(06 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(06 Marks)**

d. Reduce the following quadratic form to canonical form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ **(04 Marks)**

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