ACHARYA INSTITUTE OF TECHNOLOGY
Bangalore - 560090

CBCS Scheme

USN | 1 | A | Y | 1 | 5 | E | C | 0 | 2 | 9 |

15MAT11

First Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the n^{th} derivative of $\dfrac{x}{(x-1)^2(x+2)}$. **(06 Marks)**

b. Find the angle of intersection of the curves $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. **(05 Marks)**

c. Find the radius of curvature at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. **(05 Marks)**

OR

2 a. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that $(x^2 - 1)\, y_{n+2} + (2n+1)\, xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

b. Obtain the pedal equation of the curve $r^n = a^n \cos n\theta$. **(05 Marks)**

c. Find the derivative of arc length of $x = a\,(\cos t + \log\tan(\frac{t}{2}))$ and $y = a\sin t$. **(05 Marks)**

Module-2

3 a. Expand $\log_e x$ in powers of $(x - 1)$ and hence evaluate $\log_e(1.1)$, correct to four decimal places. **(06 Marks)**

b. If $z = \sin(ax + y) + \cos(ax - y)$, prove that $\dfrac{\partial^2 z}{\partial x^2} = a^2 \dfrac{\partial^2 z}{\partial y^2}$. **(05 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

OR

4 a. If $u(x+y) = x^2 + y^2$, then prove that $\left(\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)$. **(06 Marks)**

b. Evaluate $\underset{x\to o}{Lt}\left(\dfrac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$. **(05 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, then prove that $x\,u_x + y\,u_y + z\,u_z = 0$. **(05 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{f} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that $\vec{f}$ is irrotational. **(05 Marks)**

c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point P(2, -1, 2). **(05 Marks)**

OR

6 a. Find the directional derivative of $xy^3 + yz^3$ at (2, -1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{u} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and $\vec{v} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that $\vec{u} \times \vec{v}$ is a solenoidal vector. **(05 Marks)**

c. For any scalar field ϕ and any vector field $\vec{f}$, prove that curl $(\phi\vec{f}) = \phi$ curl $\vec{f}$ + (grad ϕ) $\times \vec{f}$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \cos^n x \, dx$, where n is a positive integer, hence evaluate $\int_0^{\pi/2} \cos^n x dx$. **(06 Marks)**

b. Solve : $(x^2 + y^2 + x)\, dx + xy dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of circles $r = 2a\cos\theta$, where 'a' is a parameter. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^{9/2}} dx$. **(06 Marks)**

b. Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(05 Marks)**

c. Water at temperature 10^0C takes 5 minutes to warm upto 20^0C in a room temperature 40^0C. Find the temperature after 20 minutes. **(05 Marks)**

Module-5

9 a. Solve the following system of equations by Gauss Elimination Method. **(06 Marks)**
$x + 2y + z = 3$, $2x + 3y + 2z = 5$, $3x - 5y + 5z = 2$.

b. Find the dominant eigen value and the corresponding eigen vector by power method $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, perform 5 iterations, taking initial eigen vector as $[1\ 1\ 1]^1$. **(05 Marks)**

c. Show that the transformation $y_1 = 2x + y + z$, $y_2 = x + y + 2z$, $y_3 = x - 2z$ is regular. Write down the inverse transformation. **(05 Marks)**

OR

10 a. Solve the following system of equations by Gauss – Seidel method. **(06 Marks)**
$10x + 2y + z = 9$, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$.

b. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

c. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. **(05 Marks)**

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