# ACHARYA INSTITUTE OF TECHNOLOGY Bangalore - 560090

# GRGS Scheme

USN

15MAT11

# First Semester B.E. Degree Examination, Dec.2016/Jan.2017 **Engineering Mathematics - I**

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Module-1

(06 Marks)

 $r = \frac{a}{1 + \cos \theta}$  and  $r = \frac{b}{1 - \cos \theta}$ .

(05 Marks)

c. Find the radius of curvature of the curve  $x^4 + y^4 = 2$  at the point (1, 1).

(05 Marks)

OR a. If x = tan(log y), find the value of  $(1+x^2)y_{n+1} + (2nx-1)y_n + (n)(n-1)y_{n-1}$ .

(06 Marks)

b. Find the Pedal equation of  $\frac{2a}{r} = 1 + \cos \theta$ .

(05 Marks)

c. Find the radius of curvature of the curve  $r^n = a^n \cos n\theta$ .

(05 Marks)

a. Explain  $\log(\cos x)$  about the point  $x = \frac{\pi}{3}$  upto 3<sup>rd</sup> degree terms using Taylor's series.

(06 Marks)

b. Evaluate  $\underset{x\to 0}{\text{Limit}} \left( \frac{\tan x}{x} \right)^{\frac{1}{x^2}}$ .

(05 Marks)

c. State Euler's theorem and use it to find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial v}$  when  $u = tan^{-1} \left( \frac{x^2 + y^2}{x + y} \right)$ .

4 a. Expand  $\frac{e^x}{1+e^x}$  using Maclaurin's series upto and including  $3^{rd}$  degree terms.

(06 Marks)

b. Find  $\frac{du}{dt}$  when  $u = x^3y^2 + x^2y^3$  with  $x = at^2$ , y = 2at. Use Partial derivatives.

(05 Marks)

c. If  $u = \frac{x_2 x_3}{x_1}$ ,  $v = \frac{x_1 x_3}{x_2}$ ,  $w = \frac{x_1 x_2}{x_3}$ , find the value of Jacobian  $J\left(\frac{u, v, w}{x_1, x_2, x_3}\right)$ .

a. A particle moves on the curve  $x = 2t^2$ ,  $y = t^2 - 4t$ , z = 3t - 5, where t is the time find the components of velocity and acceleration at time t = 1 in the direction of i - 3j + 2k.

(06 Marks)

b. Find the divergence and curl of the vector  $\vec{V} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)K$  at the point (2, -1, 1).

c. A vector field is given by  $\vec{A} = (x^2 + xy^2) i + (y^2 + x^2y)j$ , show that the field is irrotational and find the scalar potential.

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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- a. Find grad  $\phi$  when  $\phi = 3x^2y y^3z^2$  at the point (1, -2, -1). b. Find a for which f = (x + 3y)i + (y 2z)j + (x + az)k is solenoidal. (06 Marks)
  - (05 Marks) (05 Marks)
  - c. Prove that Div(curl  $\vec{V}$ ) = 0.

# Module-4

- a. Obtain the reduction formula of  $\int \sin^m x \cos^n x dx$ . (06 Marks)
  - Evaluate  $\int_{x}^{2a} x \sqrt{2ax x^2} dx$ . (05 Marks)
  - Solve  $(2x \log x xy) dy + 2y dx = 0$ . (05 Marks)

### OR

- a. Obtain the reduction formula of  $\int \cos^n x \ dx$ . (06 Marks)
  - b. Obtain the Orthogonal trajectory of the family of curves  $r^n \cos n \theta = a^n$ . Hence solve it.
  - c. A body originally at 80°C cools down at 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original?(05 Marks)

### Module-5

a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$
 (06 Marks)

b. Solve by Gauss – Jordan method the system of linear equations

2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16. (05 Marks)

c. Find the largest eigen value and the corresponding Eigen vector by power method given that

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$
 (Use  $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$  as the initial vector). (Apply 4 iterations). (05 Marks)

10 a. Use Gauss – Seidel method to solve the equations

(06 Marks)

$$20x + y - 2x = 17$$
  
 $3x + 20y - z = 18$ 

2x - 3y + 20z = 25. Carry out 2 iterations with  $x_0 = y_0 = z_0 = 0$ .

- b. Reduce the matrix  $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$  to the diagonal form. (05 Marks)
- c. Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 2yz + 2zx 2xy$  to the canonical form. (05 Marks)