USN | | | | | | | | | | 10MAT11

First Semester B.E. Degree Examination, June/July 2016
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing at least two from each part.*
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) If $y = e^{-x^2}$, then $y_{n+1} + 2xy_n + 2ny_{n-1}$ is equal to
A) n^2y B) 0 C) 2n D) 1

ii) Geometrical meaning of LMVT is that tangent parallel to
A) chord B) x – axis C) y – axis D) x = y

iii) Taylors series about the origin is
A) $y(0) + xy_1(0) + \frac{x^2}{2!}y_2(0) + --$ B) $y(0) - xy(0) + \frac{x^2}{2}y_2(0) --$
C) $xy_1(0) + \frac{x^2}{2!}y_2(0) + --$ D) $xy_1(0) - \frac{x^2}{3!} + \frac{x^5}{5!} + y_2(0) + --$

iv) Maclaurin's expansion of cos hx is
A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} --$ B) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + --$ C) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + --$ D) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} --.$

b. If y = $y = e^{a\sin^{-1}x}$, PT$(1-x^2)Y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$. **(04 Marks)**

c. State : i) Rolles theorem ii) Lagrange's mean value theorem iii) Cauchy's mean value theorem. **(06 Marks)**

d. Using the Maclaurin's series expansion, expand tan x up to the term containing x^5. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) Value of $\lt_{x\to 0} \frac{a^x - b^x}{x}$ is equal to
A) log(a/b) B) log(B/A) C) log(ab) D) 1

ii) Length of the perpendicular 'p' from the origin to the tangent
A) r cos ϕ B) r tan ϕ C) r sin ϕ D) r cot ϕ

iii) Angle between the radius vector and the tangent is
A) $\tan\phi = \frac{1}{r}\frac{d\theta}{dr}$ B) $\cot\phi = r\frac{dr}{d\theta}$ C) $\cot\phi = \frac{1}{r}\frac{d\theta}{dr}$ D) $\tan\phi = r\frac{d\theta}{dr}$

iv) Radius of curvature of a circle is
A) zero B) constant C) $\pi/2$ D) ∞.

b. Evaluate $\lt_{x\to 0}\left[2 - \frac{x}{a}\right]^{\tan\left(\frac{\pi x}{2a}\right)}$. **(04 Marks)**

c. Find the angle of intersection of curves $r = a(1 + \cos\theta)$ and $r^2 = a^2\cos 2\theta$. **(06 Marks)**

d. Prove that for the curve x = f(x) is $\rho = \frac{\left[1 + y_1^2\right]^{3/2}}{y_2}$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $u = x^y$, then u_{xy} is equal to
A) $x^{y-1}(1 + y \log x)$
B) $x^{y-1} y \log x$
C) $y^{y-1}(y + \log x)$
D) zero

ii) If $A = f_{xx}(a, b)$; $B = f_{xy}(a, b)$; $C = f_{yy}(a, b)$. Then f(x, y) will have a maximum at (a, b)
A) $f_x = 0$, $f_y = 0$, $AC - B^2 < 0$ and $A < 0$
B) $f_x = 0$, $f_y = 0$, $AC - B^2 < 0$ and $A > 0$
C) $f_x = 0$, $f_y = 0$, $AC - B^2 > 0$ and $A > 0$
D) $f_x = 0$, $f_y = 0$, $AC - B^2 > 0$ and $A < 0$

iii) If $x = r\cos\theta$, $y = r\sin\theta$, then $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ is equal to
A) 1 B) r C) 1/r D) 0

iv) $\delta x = |x - x_0|$ is referred to as
A) absolute error in x
B) error in x
C) relative error in x
D) approximate error in x.

b. If $u = e^{ax-by}\sin(ax + by)$; show that $b\dfrac{\partial u}{\partial x} - a\dfrac{\partial u}{\partial y} = 2abu$. **(04 Marks)**

c. If $u\dfrac{yz}{x}$, $v = \dfrac{zx}{y}$, $w = \dfrac{xy}{z}$; show that $\dfrac{\partial(uvw)}{\partial(xyz)} = 4$. **(06 Marks)**

d. Examine the function $xy(a - x - y)$ for extreme values. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) A vector field $\vec{F}$ is said to irrotational, if
A) div $\vec{F} = 0$ B) Curl $\vec{F} = 0$ C) Grad $\phi = F$ D) none

ii) Given $\phi = x^2y + y^2z + z^2x$, then $\nabla^2\phi$ is equal to
A) $x + y + z$
B) $2(x + y + z)$
C) $(x^2 + y^2 + z^2)$
D) $2(x - y + z)$

iii) A vector field $\vec{F}$ is said to be solenoidal, if
A) $\nabla\phi = F$ B) div $\vec{F}$ C) Curl $\vec{F}$ D) none

iv) $\nabla \times (\nabla\phi)$ is equal to
A) $\nabla^2\phi$ B) $\nabla\phi$ C) $\vec{0}$ D) 0.

b. If A and B are irrotational PT $A \times B$ is solenoidal. **(04 Marks)**

c. Show that $\nabla^2 r^n = n(n + 1) r^{n-2}$. **(06 Marks)**

d. Prove that div curl $\vec{F} = 0$. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) Value of $\int_{-\pi/2}^{\pi/2} \cos^8 x \, dx$ is equal to

A) 0 B) $\frac{35}{64}$ C) $\frac{35}{128}\pi$ D) $\frac{35}{128}$

ii) If f(x,y) = f(–x, –y) then the curve is symmetrical about

A) origin B) line y = x C) x – axis D) y – axis

iii) If f(r, θ) = f(r, π–θ) then the curve is symmetrical about

A) line $\theta = \frac{\pi}{4}$ B) $\theta = \frac{\pi}{2}$ C) $\theta = 0$ D) $\theta = \frac{\pi}{3}$

iv) Parametric equation for $x^{2/3} + y^{2/3} = a^{2/3}$ (Astroid) is

A) $x = a\cos^3\theta, y = a\sin^3\theta$ B) $x = a\cos^2\theta, y = a\sin^2\theta$

C) $x = \cos^3\theta, y = \sin^3\theta$ D) none.

b. Given : $\int_0^{\pi} \frac{dx}{a + b\cos x} = \frac{\pi}{\sqrt{a^2 + b^2}} (a > b)$; evaluate $\int_0^{\pi} \frac{dx}{(a + b\cos\theta)^2}$. **(04 Marks)**

c. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n\theta \, d\theta$. **(06 Marks)**

d. Find the area of the cardiod r = a (1 – cos θ) **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) Substitution that transformations the equation : $\frac{dy}{dx} = \frac{x + y + 1}{2x + 2y + 3}$ to homogeneous form is

A) x – y = t B) (y/x) = t C) x/y = t D) x + y = t

ii) Integrating factor for the D.E. $\frac{dy}{dx} + Py = Q$, where P and Q are function x only

A) $e^{\int p\, dy}$ B) $\int P\, dx$ C) $\int P\, dy$ D) $e^{\int P\, dx}$

iii) Necessary and sufficient condition for the DE M(x, y) dx + N(x, y)dy = 0 to be an exact is

A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ B) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial y} = 0$ C) $\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$ D) none

iv) The family of straight lines passing through the origin is represented by the differential equation :

A) y dx + x dy = 0 B) x dx + y dy = 0 C) x dy – y dx = 0 D) none.

b. Solve $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$. **(04 Marks)**

c. Solve $(1 + y^2)dx = (\tan^{-1}y - x)\, dy$. **(06 Marks)**

d. ST the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(06 Marks)**

7 a. Choose the correct answers for the following : (04 Marks)

i) Rank of $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ is

A) 2 B) 1 C) –3 D) 4

ii) A set of 'm' linear equations, with n unknowns posses infinite solution if

A) $\rho(A) = \rho[A : B] = r = n$ B) $\rho(A) = \rho[A : B] = r < n$

C) $\rho(A) \neq \rho[A : B]$ D) $\rho(A) = \rho[A : B] = r > n$

iii) For a system of linear homogeneous equation if $\rho(A) = \rho[A : B] = n$, where n is the number of un known system was

A) trivial solution B) non trivial solution C) both A and B D) no solution

iv) For non homogeneous system of linear equations, Gauss lamination method is applicable, of the coefficient matrix is reduced to

A) Symmetric matrix B) lower triangular matrix

C) diagonal matrix D) upper triangular matrix.

b. Using the elementary transformation reduce the matrix A to Echelon form $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

Find the rank of the matrix A. (04 Marks)

c. Investigate the values of λ and μ so that the equations : $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, have i) no solution ii) unique solution iii) infinite number of solutions. (06 Marks)

d. Solve the system of equations by Gauss Jordon method : $x + y + z = 9$, $x - 2y + 3z = 8$ $2x + y - z = 3$. (06 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) Linear transformation $Y = AX$ is regular, if

A) A is singular B) A is square C) A is non singular D) none

ii) Sum of eigen values of a square matrix is equal to

A) Sum of the principle diagonal elements

B) product of principle diagonal elements

C) determine of value of that matrix

D) none

iii) The matrix B of same order as A is said to be similar if these exist D such that

A) $A = P^{-1}BP$ B) $B = P^{-1} AP$ C) $A^n = PD^n P^{-1}$ D) $B = P^{-1} DP$

iv) Matrix 'D' which diagonalises 'A' is

A) Spectral matrix of A B) Orthogonal matrix of A

C) null matrix D) modal matrix of A.

b. Show that the transformation, $y_1 = 2x_1 x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$, is Regular and write down the inverse transformation. (04 Marks)

c. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form. (06 Marks)

d. Find the rank, index, signature of the following quadratic form :
$2x^2 - 2y^2 + 2y^2 - 2xy - 8yz + 6zx$. (06 Marks)

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