USN | | | | | | | | | | 13MCA12

First Semester MCA Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define : i) Principle of duality ii) Tautology iii) Contradiction. **(06 Marks)**

b. Prove that for any three propositions p, q and r, the compound propositions

i) $[((\neg q) \wedge (p \rightarrow q)] \rightarrow (\neg p)$

ii) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$.

are tautologies using truth tables. **(06 Marks)**

c. Prove the following logical equivalences using laws of logic.

i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

ii) $[p \vee q \vee (\sim p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$

iii) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$. **(08 Marks)**

2 a. Prove the following logical equivalences using truth tables :

i) $[p \wedge (p \rightarrow q) \wedge r] \Rightarrow [(p \vee q) \rightarrow r]$

ii) $\{[p \vee (q \vee r)] \wedge \neg q\} \Rightarrow p \vee r$. **(06 Marks)**

b. Test whether the following arguments are valid.

i)
$$\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

ii)
$$\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ \neg q \vee \neg s \\ \hline \therefore \neg(p \wedge r). \end{array}$$
(08 Marks)

c. Define : i) open statement ii) universal and existential quantifiers with an example for each. **(06 Marks)**

3 a. Give : i) direct proof ii) indirect proof iii) proof by contradiction for the following statement. "If n is an odd integer, then n + 9 is an even integer". **(06 Marks)**

b. State and prove distributive laws. **(06 Marks)**

c. Among the integers from 1 to 200, find the number of integers that are :
i) not divisible by 5 ii) divisible by 2 or 5 or 9 iii) not divisible by 2 or 5 (or) 9. **(06 Marks)**

d. Find out the member of arrangements of no adjacent letter "A" in "TALLAHASEE". **(02 Marks)**

4 a. What are the two steps involved in induction principle? Prove that :

$$1^2 + 3^2 + 5^2 + 7^2 + \text{- - - -} 2(n-1)^2 = 1/3^n(2n-1)(2n+1).$$ **(06 Marks)**

b. Solve the recurrence relation :

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n-2}}{2^n}.$$ **(08 Marks)**

c. Suppose $\cup$ is an universal set, and $= A_1, A_2, \text{- - - - -} A_n \subseteq U$. Prove that $\overline{A_1 \cap A_2 \cap \ldots\ldots A_n} = \overline{A_1} \cup \overline{A_2} \ldots\ldots \cup \overline{A_n}$ using mathematical induction. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

5 a. Let A = {1, 2, 3} and B = {2, 4, 5}. Determine the following :
   i) $A \times B$
   ii) number of relations from A to B
   iii) number of relations form A to B that contain (1, 2) and (1, 5)
   iv) number of binary relation on A. **(06 Marks)**

b. Let R and S be relations on set A. Prove that :
   i) If R and S are reflexive so are $R \cap S$ and $R \cup S$
   ii) If R and S are symmetric, so are $R \cap S$ and $R \cup S$
   iii) If R and S are antisymmetric, so are $R \cap S$. **(06 Marks)**

c. Define a Poset. For the given set A, draw the digraph. Verify whether (A, R) is a Poset and draw the Hasse diagram of the same A = {1, 2, 3, 4}. **(08 Marks)**

6 a. Find the least number of ways of choosing three different numbers from 1 to 10 so that all choices have the same sum. **(04 Marks)**

b. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$

   i) Determine f(0), $f^{-1}(0)$, f(5/3), f(–3)
   ii) Find $f^{-1}([-5, 5])$, $f^{-1}([-6, 5])$. **(07 Marks)**

c. Let A = {1, 2, 3} and B = {1, 2, 3, 4, 5}. Find whether the following functions are :
   i) one to one ii) onto.
   i) f = {(1, 1) (2, 3) (3, 4)}
   ii) g = {(1, 1), (2, 2) (3, 3)}. **(03 Marks)**

d. Find the number of ways of distributing 4 distinct objects among 3 identical containers with some containers empty. **(06 Marks)**

7 a. Define : i) graph ii) sub graph iii) compliment of a graph iv) bipartite graph.. **(06 Marks)**

b. Define chromatic member of a graph. Prove that the chromate member of any connected biparatite graph with atleast two vertices is 2. **(06 Marks)**

c. Find the shortest path from vertex a to e in the weighted graph using Dijksta's algorithm and show the steps in detail. **(08 Marks)**

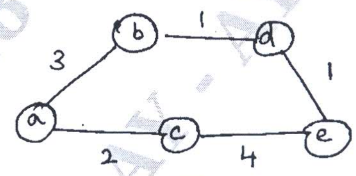

Fig.Q7(c)

8 a. Define a tree. What is the maximum height a tree with n vertices can attain, if :
   i) it's a binary tree ii) a–complete binary tree? **(06 Marks)**

b. Apply merge sort on 6, 2, 7, 3, 4, 9, 5, 1, 8 and explain the steps in detail. **(06 Marks)**

c. List out the i) pre order ii) post order iii) inorder of the given tree. **(08 Marks)**

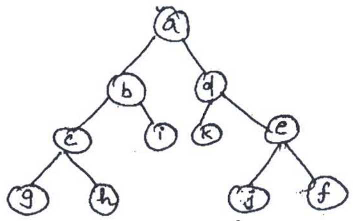

Fig.Q8(b)