USN



# Sixth Semester B.E. Degree Examination, Dec.2018/Jan.2019
## Finite Element Methods

Time: 3 hrs. Max. Marks:100

**Note:1.** ***Answer FIVE full questions, selecting at least TWO questions from each part.***
**2.** ***No hand book required.***

## PART – A

1 a. Explain the step-by-step procedure for the application of finite element method for structural problems. **(08 Marks)**

b. Derive the differential equations of equilibrium for a body subjected to a three dimensional stress system and body force. **(12 Marks)**

2 a. Derive the Euler-Lagrange's equation by considering the functional,

$$I = \int_{x_1}^{x_2} F(x, u, u', u'')dx$$

Also obtain the expressions for natural boundary conditions and essential boundary conditions. **(10 Marks)**

b. A system of springs is subjected to a force of 500 N as shown in Fig. Q2 (b). Determine the forces at points A and D, and displacement at points B and C. Apply the principle of minimum potential energy. **(10 Marks)**

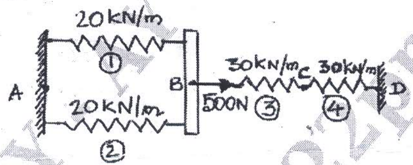


Fig. Q2 (b)

3 a. For a constant strain triangular element (CST), derive (i) Shape functions in natural coordinates and draw the sketches showing distributions of shape functions, and (ii) Jacobian matrix. **(14 Marks)**

b. Compute the shape functions $N_1$, $N_2$ and $N_3$ at point P(8, 7) for the constant strain triangular element shown in Fig. Q3 (b). **(06 Marks)**

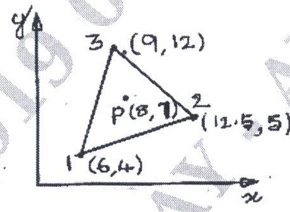


Fig. Q3 (b)

4 a. List three properties each of, (i) Shape functions and (ii) Stiffness matrix. **(06 Marks)**

b. A stepped bar with it's both ends fixed as shown in Fig. Q4 (b) is subjected to an increase in temperature of 50°C. Determine displacements and forces at points A, B, C and D. Take Young's moduli of steel, bronze and aluminium as $E_S = 200$ GPa, $E_B = 83$ GPa and $E_{AL} = 70$ GPa respectively. The coefficients of thermal expansions are $\alpha_S = 12\times10^{-6}/°C$, $\alpha_B = 19\times10^{-6}/°C$ and $\alpha_{AL} = 22\times10^{-6}/°C$. **(14 Marks)**

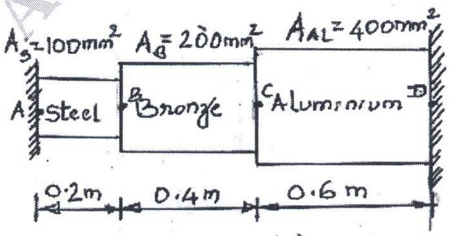


Fig. Q4 (b)

## PART – B

5 a. Derive Lagrange shape functions for a bar element with cubic displacement distribution and show their distributions with neat sketches. **(12 Marks)**

b. Use two point Gaussian quadrature to evaluate the following integral:

$$I = \int_{-1}^{1}\int_{-1}^{1}(\xi^3 - 1)(\eta - 1)^2\, d\xi d\eta .$$ **(08 Marks)**

6 a. Derive the expression for the stress induced in a truss member. Start from the expression for the stress ($\sigma$), which is a function of strain-displacement matrix, transformation matrix and displacement vector for the element. **(06 Marks)**

b. A truss is subjected to a force of 10 KN as shown in Fig. Q6 (b). Determine (i) Displacements and forces at points A, B and C (ii) Local forces on the member AB and (iii) Stress induced in the member BC. Take Young's modulus for the material of the truss as E = 210 GPa and cross sectional area of each truss member as $A = 600\ mm^2$. **(14 Marks)**

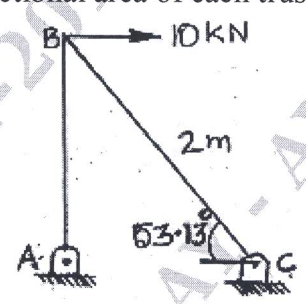


Fig. Q6 (b)

7 a. Derive the statically equivalent nodal force vector for a two noded beam element subjected uniformly distributed load (W). **(08 Marks)**

b. A beam with fixed end and roller support is subjected to a point force of 20 KN as shown in Fig. Q7 (b). Determine (i) Transverse forces and bending moments, and (ii) Deflections and slopes at points A, B and C. Take $E = 2\times10^8\ KN/m^2$ and $I = 8\times10^{-6}\ m^4$. **(12 Marks)**

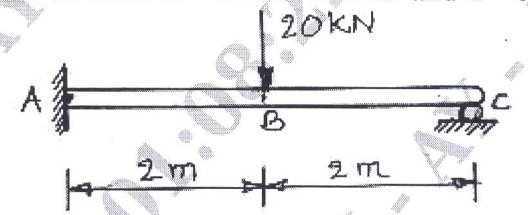


Fig. Q7 (b)

8 a. Explain the specified temperature and specified heat flux boundary conditions, with neat sketches. **(06 Marks)**

b. Determine the temperature distribution through the composite wall subjected to convection heat loss on the right side surface with convective heat transfer coefficient as shown in Fig. Q8 (b). The ambient temperature is $-5^\circ C$.
Consider area (A) of the wall to be $1\ m^2$. **(14 Marks)**

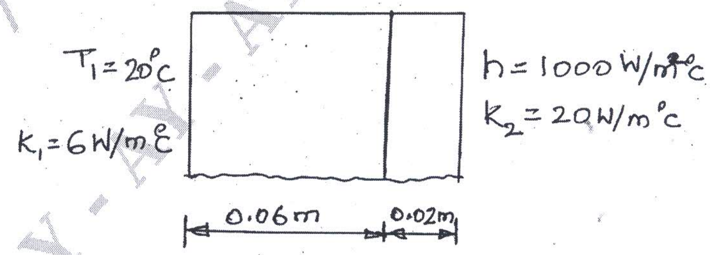


Fig. Q8 (b)

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