USN | | | | | | | | | | 10CS42

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Define : (i) Complete graph (ii) Induced Subgraph (ii) Euler's circuit. Give one example for each. **(05 Marks)**

b. Show that there is no graph with 12 vertices and 28 edges where
   i) The degree of each vertex is either 3 or 4
   ii) The degree of each vertex is either 3 or 6 **(05 Marks)**

c. Define isomorphism of two graphs. By labeling the graphs shows that two graphs are isomorphic.

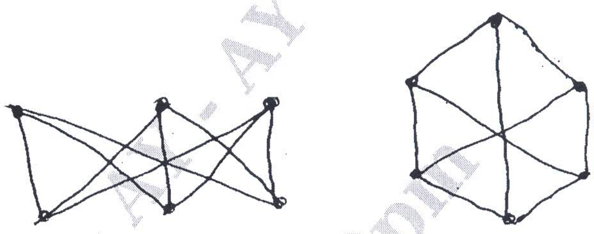

Fig Q1(c) **(05 Marks)**

d. Let G = (V, E) be the undirected graph in Fig Q1(d) How many paths are there in G from a to h? How many of these paths have a length 5?

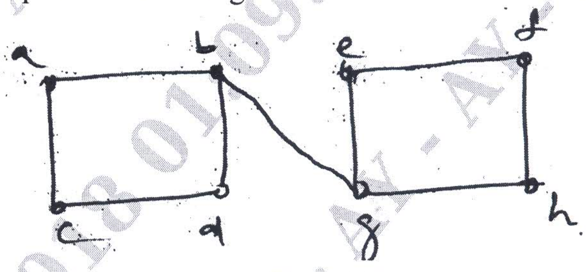

Fig Q1(d) **(05 Marks)**

2 a. A connected planar graph G with n vertices and m edges has exactly m – n + 2 regions in all of its diagrams. **(07 Marks)**

b. If 4 colours are used, find in how many ways can this graph be properly colured? Hence find the chromatic number (Refer Fig Q2(b)

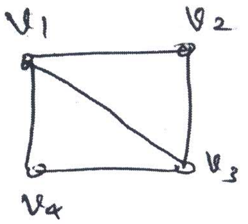

Fig Q2(b) **(07 Marks)**

c. Consider the graph K_2, 3 shown below, Let λ devote the number of colours available to properly colour the vertices of this graph find
   i) How many proper colouring of the graph have vertices a, b coloured same
   ii) How many proper colourings of the graph have vertices a, b coloured differently.
   iii) The cheomatic polynomial of the graph.

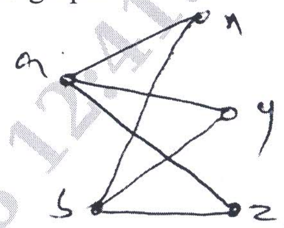

Fig Q2(c) **(06 Marks)**

3 a. Define a binary rooted tree and show that a tree with n vertices has n – 1 edge. **(07 Marks)**
b. Obtain an optimal prefix code for the message LETTER RECEIVED Indirect the code. **(07 Marks)**
c. Define: i) Weighted Tree ii) Prefix codes iii) Optimal prefix code. **(06 Marks)**

4 a. Explain Prim's Algorihtm and find a minimal spanning tree for the weighted graph show below

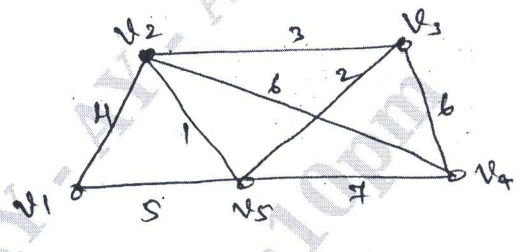

Fig Q4(a) **(06 Marks)**

b. State and prove maximum flow and minimum cut theorem. Also find the maximum flow from the vertices A and vertex Z in the network shown below

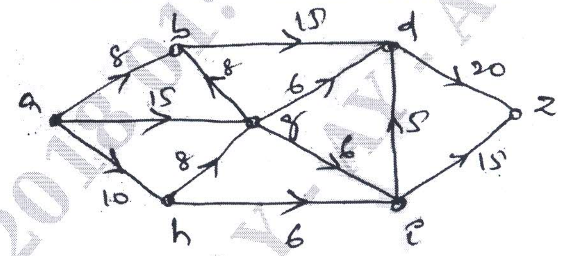

Fig Q4(b) **(07 Marks)**

c. Using the Dijkstras algorithm, obtain the shortest path from vertex 1 to each of the other vertices in the weighted, directed network shown below indicate the weight of these shortest paths.

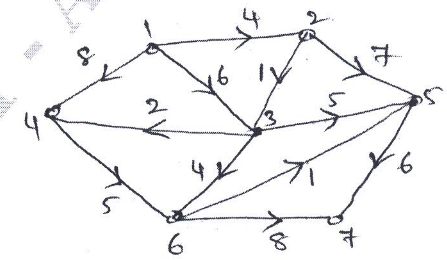

Fig Q4(c) **(07 Marks)**

PART – B

5 a. How many arrangement are the for all letters in the world SOCIOLOGICAL? In how many of these arrangement (i) A and G are adjacent (ii0 All the vowels are adjacent. **(07 Marks)**

b. Determine the coefficient of $x^2y^2z^3$ the expansion of $(3x - 2y - 4z)^7$ **(06 Marks)**

c. Using Catalan number find the possible way of arranging four 1's and four 0's such that in each arrangement the number of 0's never exceed the number of 1's. **(07 Marks)**

6 a. In how many ways can one distribute eight identical balls into four destined containers so that

i) No container is left empty?

ii) The fourth container gets and odd number of balls? **(06 Marks)**

b. In how many ways can the arrange the letter in the word CORRESPONDENTS so that

i) There is no pair of consecutive identical letters?

ii) There are exactly two pairs of consecutive identical letters?

iii) There are at least three pairs of consecutive identical letters? **(07 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed for four boys B_1, B_2, B_3, B_4. The boys B_1, B_2 do not wish to have apple, the boy B_3 does not want banana or mange and B_4 refuses orange. In how many ways the distribution can be made so that no boy is disposed? **(07 Marks)**

7 a. Find a generating function for each of the following sequence

i) 1, 1, 0, 1, 1, 1 . . . . . . ii) 0, 2, 6, 12, 20, 30, 42, ... **(06 Marks)**

b. A bag contains a large number of red, green, white and black marbles, with at least 24 of each colour in how many ways can one select 24 of these marbles. So that there are even number of white marbles and at least six blacks marbles? **(07 Marks)**

c. A ship carries 48 flags, 12 each of the colours, red white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships.

i) How many of these signals use an even number of blue flage and an odd number of black flags?

ii) How many of the signals have at least three white flags or no white flag at all? **(07 Marks)**

8 a. The number of viruses affected files in a system is 1000 (to start with) and this increase 250% every two hours. Use a recurrence relation to determine the number of viruses affected files in the system after one day. **(06 Marks)**

b. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$, and $a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$. Determine the constants b and c and then solve the relation for a_n. **(07 Marks)**

c. Solve the recurrence relation

$a_{n+2} - 4a_{n+1} + 3a_n = -200,\ n \geq 0$

$a_0 = 3000,\ a_1 = 3300$ **(07 Marks)**

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