



Second Semester B.E./B.Tech. Degree Examination, June/July 2026 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Evaluate : $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y \, dx \, dy$ (06 Marks)
- b. Evaluate by change of order of integration
 $\int_0^1 \int_x^1 \frac{x}{\sqrt{x^2 + y^2}} \, dy \, dx$ (07 Marks)
- c. Show that $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ (07 Marks)

OR

- 2 a. Evaluate : $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$ (06 Marks)
- b. Find the area of the circle $x^2 + y^2 = a^2$ by double integration. (07 Marks)
- c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} \, d\theta = \pi$ (07 Marks)

Module-2

- 3 a. Find the directional derivative of the function $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along the direction $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ (06 Marks)
- b. If $\vec{A} = xz^3\mathbf{i} - 2x^2yz\mathbf{j} + 2yz^4\mathbf{k}$ find $\nabla \cdot \vec{A}$ ($\text{div } \vec{A}$) and $\nabla \times \vec{A}$ ($\text{curl } \vec{A}$) at $(1, -1, 1)$. (07 Marks)
- c. Show that $\vec{F} = (2xy^2 + yz)\mathbf{i} + (2x^2y + xz + 2yz^2)\mathbf{j} + (2y^2z + xy)\mathbf{k}$ is irrotational. Find a scalar function ϕ such that $\vec{F} = \nabla\phi$ (07 Marks)

OR

- 4 a. If $\vec{F} = x^2\mathbf{i} + xy\mathbf{j}$ evaluate $\int_c \vec{F} \cdot d\vec{r}$ from (0, 0) to (1, 1) along
 i) the line $y = x$
 ii) the parabola $y = \sqrt{x}$ (06 Marks)
- b. Using Greens theorem, evaluate $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the closed curve formed by $y = \sqrt{x}$ and $y = x^2$. (07 Marks)
- c. Using Stokes theorem, evaluate $\int_s (\text{curl } \vec{F}) \cdot \hat{n} \cdot ds$ for $\vec{F} = (x^2 + y^2)\mathbf{i} - 2xy\mathbf{j}$ taken round the rectangle bounded by $x = 0$, $x = 2a$, $y = 0$ and $y = 2b$. (07 Marks)

Module-3

- 5 a. Form a partial differential equation by eliminating arbitrary constant from $ax^2 + by^2 + z^2 = 1$ (06 Marks)
- b. Solve : $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$ (07 Marks)
- c. Derive one dimensional wave equation. (07 Marks)

OR

- 6 a. Form partial differential equation by eliminating the arbitrary function from $z = e^{ax + by}$ and $(ax - by)$ (06 Marks)
- b. Solve : $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y} + a$ (07 Marks)
- c. Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$ if $y = (2n+1)\frac{\pi}{2}$ (07 Marks)

Module-4

- 7 a. Find a real root of the equation $2x - \log_{10} x = 7$ in (3.5, 4) by Regula-Falsi method. (06 Marks)
- b. Find $y(1.4)$ using Newton's forward interpolation formula given that
- | | | | | | |
|---|----|----|----|-----|-----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 10 | 26 | 58 | 112 | 194 |
- (07 Marks)
- c. Evaluate : $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $1/3^{\text{rd}}$ rule taking four equal strips. (07 Marks)

OR

- 8 a. Find the real root of the equation $xe^x - 2 = 0$ correct to three decimal places using Newton-Raphson method. (06 Marks)
- b. Determine $f(x)$ as a polynomial in x for the following data using Newtons divided difference formula

x	2	4	9	10
y	4	56	711	980

(07 Marks)

- c. Use Simpsons $3/8^{\text{th}}$ rule to evaluate $\int_1^4 e^{1/x} dx$ by considering 4 ordinates (3 equal parts).

(07 Marks)

Module-5

- 9 a. Find y at $x = 0.1$ correct to four decimal places. Given $dy/dx = x-y^2$ and $y = 1$ at $x = 0$ apply Taylor's series method. (06 Marks)
- b. Find the value of y at $x = 1.1$. Given that $\frac{dy}{dx} + y = 2x$; $y = 3$ at $x = 1$ by using Runge-Kutta method of order 4 ($h = 0.1$). (07 Marks)
- c. Find $y(4.4)$ using Milne's predictor corrector method given $5xy' + y^2 - 2 = 0$; $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$. (07 Marks)

OR

- 10 a. Use Taylor's series method to find y at $x = 0.1$ given $dy/dx - 2y = 3e^x$; $y(0) = 0$ upto the term containing x^3 . (06 Marks)
- b. Using modified Euler's method find y at $x = 0.2$ given $dy/dx = 3x + y/2$ with $y(0) = 1$ taking $h = 0.1$. (07 Marks)
- c. Given $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0.4) = 1$, find $y(0.5)$ taking $h = 0.1$ using Runge-Kutta 4^{th} order method. (07 Marks)
