

First Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026

Calculus and Linear Algebra : CSE Stream

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. M : Marks , L: Bloom's level , C: Course outcomes.

3. VTU Handbook is permitted.

Module – 1				M	L	C
Q.1	a.	Find the total derivative when $u = x^3y^2 + x^2y^3$ where $x = at^2, y = 2at$		6	L2	CO1
	b.	If $u = \log(\tan x + \tan y + \tan z)$ then, show that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$		7	L2	CO1
	c.	Expand $e^x \sin y$ in the powers of x and y as far as terms of third term using Maclaurin's series.		7	L3	CO1
OR						
Q.2	a.	If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$. Evaluate the Jacobian of (u, v, w) with respect to (x, y, z) .		6	L2	CO1
	b.	If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$		7	L2	CO1
	c.	Examine the function for the extreme values, given $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$		7	L3	CO1
Module – 2						
Q.3	a.	Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.		6	L2	CO1
	b.	Show that $\vec{F} = (y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ is irrotational and find the scalar potential.		7	L2	CO1
	c.	With usual notation, Prove that cylindrical polar coordinate system is orthogonal.		7	L3	CO1
OR						
Q.4	a.	Given $\vec{F} = \nabla(xy^3z^2)$, Find $\text{div}(\vec{F})$ and $\text{curl}(\vec{F})$ at $(1, -1, 1)$.		6	L2	CO1
	b.	Show that $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational.		7	L2	CO1
	c.	Express $2y\hat{i} - z\hat{j} + 3x\hat{k}$ in terms of spherical polar coordinates.		7	L3	CO1

Module – 3

Q.5	a.	Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & 2 & 4 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$	6	L2	CO2
	b.	Using Gauss Jordan method, solve the system of equations $x + y + z = 9, x - 2y + 3z = 8, 2x + y - z = 3.$	7	L2	CO2
	c.	Diagonalize the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$.	7	L3	CO2

OR

Q.6	a.	Investigate the value of μ and λ such that the equations, $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$, may have (i) unique solution (ii) infinite solution (iii) no solution	6	L2	CO2
	b.	Using Gauss Elimination method, solve the system of equations $x + 2y + z = 3, 2x + 3y + 3z = 10, 3x - y + 2z = 13$	7	L2	CO2
	c.	Find the eigen value and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	7	L3	CO2

Module – 4

Q.7	a.	Determine whether the vectors $(8,0,5)$ is a linear combination of the vectors $(1,2,3), (0,1,4), (2,-1,1)$	6	L2	CO3
	b.	Find the basis and dimension of the subspace spanned by the vectors $(2,4,2), (1,-1,0), (1,2,1), (0,3,1)$ in $V_3(R)$.	7	L2	CO3
	c.	Find the basis and dimension of the row space, column space and null space of the matrix $\begin{bmatrix} 1 & -1 & 1 & 3 & 2 \\ 2 & -1 & 1 & 5 & 1 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix}$.	7	L3	CO3

OR

Q.8	a.	Find the coordinates of the vector $v = (0,1,3)$ with respect to the basis $B = \{(1,1,0), (0,1,1), (1,0,1)\}$.	6	L2	CO3
	b.	Define inner product space. Given $u = (1,2,4), v = (2,-3,5), w = (4,2,-3)$ in R^3 . Find (i) $\langle u, v \rangle$ (ii) $\langle v, w \rangle$ (iii) $\langle u, w \rangle$ (iv) $\ u\ $ (v) $\ v\ $	7	L2	CO3
	c.	What is a subspace? Prove that the subset $W = \{(x,y,z)/x - 3y + 4z = 0\}$ of the vector space R^3 is a subspace.	7	L3	CO3

Module – 5

Q.9	a.	Show that the transformation $T: R^2 \rightarrow R^3$ defined by $T(x,y) = (x+y, x-y, y)$ is a linear transformation.	6	L2	CO3
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	b.	Prove that the transformation $T: R^2 \rightarrow R^2$ is singular and find its kernel, if the transformation is given by $T(x, y) = (2x + 4y, x + 2y)$.	7	L2	CO3
	c.	Verify Rank- Nullity theorem for the transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y, y - z, x + 2z)$	7	L3	CO3
OR					
Q.10	a.	Find the matrix representing the linear transformation $T: R^3 \rightarrow R^4$ defined by $T(x, y, z) = (x + y + z, 2x + z, 2y - z, 6y)$	6	L2	CO3
	b.	If $T: R^3 \rightarrow R^3$ is a linear transformation defined by $T(x, y, z) = (x + z, x - z, y)$ Show that T is invertible and find T^{-1}	7	L2	CO3
	c.	Verify Rank- Nullity theorem for the transformation $T: R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x + y, z)$	7	L3	CO3
